

# Probability Density Functions for Channel Holding Times under Cellular CDMA Soft Handoffs: Blocking Probability Approach

## บทคัดย่อ

ในระบบเซลลูลาร์ซีดีเอ็มเอ (CDMA: Code Division Multiple Access) ดัชนีสมรรถนะได้แก่ ความน่าจะเป็นการบล็อก คุณภาพของการบริการ ประสิทธิภาพทรัพยากรทรังก์ และอื่น ๆ ได้นำมาใช้เพื่อประเมินระบบ บทความนี้นำเสนอฟังก์ชันความหนาแน่นความน่าจะเป็น (pdfs: probability density functions) สำหรับเวลายึดช่องสัญญาณ บนพื้นฐานแบบจำลองเรขาคณิตซอฟต์แวร์แฮนด์ออฟ pdfs ที่ได้ในแบบจำลองที่นำเสนอเป็นพื้นฐานในการหาค่าดัชนีสมรรถนะข้างต้น นอกจากนี้วิธีหาความน่าจะเป็นการบล็อกในซอฟต์แวร์แฮนด์ออฟ ซึ่งใช้ pdfs ที่ได้ร่วมกับสมการสมดุลภาระและสูตรเออร์แลงบีได้มีการนำเสนอไว้ด้วย

คำสำคัญ - เซลลูลาร์ ซีดีเอ็มเอ ฟังก์ชันความหนาแน่นความน่าจะเป็น เวลายึดช่องสัญญาณ แฮนด์ออฟ ความน่าจะเป็นการบล็อก สมการสมดุลภาระ

## Abstract

In cellular CDMA (Code Division Multiple Access) system, the performance indicators such as blocking probability, quality of service, trunk-resource efficiency, and so on, are used to evaluate the system. This paper presents the probability density functions (pdfs) for channel holding times based on the proposed soft handoff geometric model. The derived pdfs in the proposed model are the fundamentals for obtaining the performance indicators above. In addition, the method to get blocking probability in soft handoff using derived pdfs with load balance equation and Erlang-B formula is illustrated.

Index Terms - Cellular, CDMA, probability density function, channel holding time, handoff, blocking probability, load balance equation.

# Probability Density Functions for Channel Holding Times under Cellular CDMA Soft Handoffs: Blocking Probability Approach

\*Bongkarn Homnan

Normally, handoff is used to maintain the communication between Mobile Stations (MSs). The handoff MS as shown in Fig. 1 uses new resource such as frequency, channel, or code from new Base Station (BS) [1]. For Hard HandOff (HHO), is used in cellular FDMA (Frequency Division Multiple Access) and TDMA (Time Division Multiple Access) systems, the MS will break an old link from old resource (BS) before making new link from new resource (BS), 'break before make' process [2]. On the other hand, Soft HandOff (SHO) is the 'make before break' process. That is, an MS uses the resources from more than one BS

simultaneously before releasing resource from the old BS. Therefore, during SHO, the quality of traffic channel is improved with multiple links.

One of the essential fundamentals for analyzing the handoff performance in cellular communication systems is the probability density function (pdf) for channel holding time of traffic channel [3] and for mobility including pdf of MS's velocity and pdf of MS's direction [3], [4], [5], [6]. All pdfs above are used to obtain the performance indicators such as blocking probability, quality of service, trunk-resource efficiency, and so on [3].

This paper is organized as follows. Section II briefly explains the

---

\*Associate Professor, Department of Telecommunications Engineering,  
Dhurakij Pundit University Bangkok, Thailand bongkarn@dpu.ac.th

fundamental of SHO. Section III concentrates on the proposed geometric model developed from [3] especially in the overlap area between 3 cells. Section IV applied the load balance equation for SHO. Section V expressed the pdfs for channel holding times using the

proposed geometric model. Section VI describes the performance indicators and computes the results from load balance equation in section IV and derived pdfs in section V with Erlang-B formula for getting the blocking probability (PB).

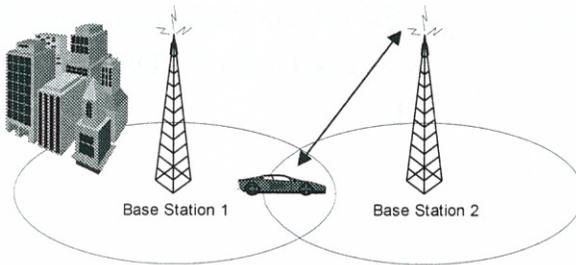


Fig. 1. Handoff.

## II. SOFT HANDOFF

In SHO process, an MS acts as the assistant of the BS by measuring and reporting the signal strength of received pilots to the BS. There are channel lists [2], [7] for MS including the members of which include the Active Set (AS), the Candidate Set (CS), the Neighbor Set (NS), and the Remaining Set (RS). The AS contains the currently used channel(s). The CS contains channels, the qualities of which are almost as good as those in AS, and one of them can be chosen for SHO as a new member of AS. The NS is the set of channels which is not included in the AS and CS but is reasonably strong. The RS contains other

channels that are not the members of all other sets. While an MS handoff, it searches for other usable cells. If the NS pilot strength becomes above add threshold ( $T\_ADD$ ), the MS includes the pilot into the CS and removes it from the NS. If the current AS pilot strength decreases below drop threshold ( $T\_DROP$ ) for more than  $T\_TDROP$  seconds, the MS moves the pilot from the AS into the NS.

## III. PROPOSED GEOMETRIC MODEL

For analysis of the cellular CDMA system, there are proposed distances as follows:

1) cell Distance ( $D$ ) is the distance from BS to the corner of hexagonal cell as shown in Fig. 2.

2) equivalent  $D$  ( $D_{eq}$ ) is the radius of circle (circular cell) which occupies the same area as hexagonal cell in 1).

3) adding Distance ( $D_a$ ) is the distance that MS uses to add pilot signal which is above  $T_{ADD}$ , of target cell to its AS list.

4) equivalent  $D_a$  ( $D_{aeq}$ ) is the radius of circular cell which occupies the same area as hexagonal cell in 3).

5) dropping Distance ( $D_d$ ) is the distance MS uses to start timer of low pilot signal which is below  $T_{DROP}$ .

6) time-based drop Distance ( $D_{td}$ ) is the distance from BS to the point that MS drops low pilot signal when the timer is finished with  $T_{TDROP}$  seconds [2], [7].

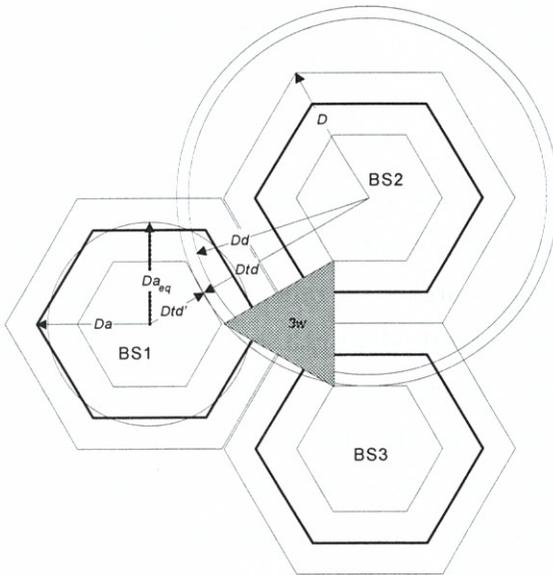


Fig. 2. Proposed geometric model

In addition, the proposed geometric cells consist of three areas as follows:

- 1) Inner Cell Area (ICA)
- 2) Normal Cell Area (NCA)
- 3) Outer Cell Area (OCA)

ICA is a circular cell with radius  $D_{aeq}$  expressed in equation (1). ICA in the model is the area where MS uses only one link (1-way handoff).

$$ICA = \pi Da_{eq}^2 \quad (1)$$

Then,  $Da_{eq}$  can be rewritten as

$$Da_{eq} = \sqrt{\frac{3\sqrt{3}}{2\pi}} Da \quad (2)$$

In the same way,  $D_{eq}$  can be expressed as

$$D_{eq} = \sqrt{\frac{3\sqrt{3}}{2\pi}} D \quad (3)$$

NCA is the hexagonal cell with parameter  $D$  and OCA is the ring-shape area as expressed in equation (4).

$$OCA = \pi(D^2d^2 - Da_{eq}^2) \quad (4)$$

Note:  $3w$  in Fig. 3 represents 3-way handoff area. Thus, the area besides 1-way handoff area and 3-way handoff area is 2-way handoff area.

#### IV. LOAD BALANCED EQUATION FOR SOFT HANDOFF USING PROPOSED GEOMETRIC MODEL

As considering flow control of traffic load, there are incoming and outgoing MSs in each cell all the time [3], [8]. Therefore, at steady state, for uniform traffic load condition, it is assumed that the traffic load in any cell is constant (load balance). The load balance equation for SHO according to any area in proposed geometric model can be expressed in equation (5)

$$\{\lambda_{ICA}E[T_{hICA}] + \lambda_{OCA}E[T_{hOCA}] + \lambda_{OCA'}E[T_{hOCA'}]\}(1 - P_B) = (\lambda_{ICA} + \lambda_{OCA})E[T](1 - P_B) + \lambda_{OCA'}E[T]P_B \quad (5)$$

where

$P_B$  is the blocking probability.

$\lambda_{ICA}$  is the arrival rate of new calls in ICA occurs according to independent Poisson process.

$\lambda_{OCA}$  is the arrival rate of handoff calls from OCA occurs according to independent Poisson process.

$\lambda_{OCA'}$  is the handoff rate enters the cell from outside OCA according to an independent Poisson process.

$T$  is call duration time ( $T=1/\text{call departure rate} (?)$ ).

$T_{hICA}$  is the channel holding time for a new call originated in ICA.

$T_{hOCA}$  is the channel holding time for a handoff call originated from OCA.

$T_{hOCA'}$  is the channel holding time for a handoff call originated from other cell.

$E[T]$  is the expectations of call duration time.

$E[T_{hx}]$  can be calculated as follow [8].

$$E[T_{hx}] = \int_0^{\infty} t [e^{-\mu} f_{T_x}(t) + \{1 - F_{T_x}(t)\} \mu e^{-\mu}] dt = \frac{1}{\mu} - \int_0^{\infty} \frac{1}{\mu} e^{-\mu} f_{T_x}(t) dt = \{1 - \Pr(T > T_x)\} E[T] \quad (6)$$

$X$  can be substituted by ICA, OCA, or OCA' ( $T_x$  is the dwell time of MS in  $X$  area) therefore  $T_x$  can be  $T_{ICA}$ ,  $T_{OCA}$  or  $T_{OCA'}$ , respectively. Note  $f_{T_x}$  is the pdf of  $T_x$  and  $F_{T_x}$  is the cumulative density function (cdf) of  $T_x$ . Due to the uniform traffic in geometric model in Fig. 2,  $\lambda_{ICA}$  and  $\lambda_{OCA}$  can be obtained as expressed in equations (7) and (8).

$$\lambda_{ICA} = (ICA / NCA) \lambda_{NCA} \quad (7)$$

and

$$\lambda_{OCA} = (OCA / NCA) \lambda_{NCA} \quad (8)$$

In the next section, pdfs of  $T_{ICA}$ ,  $T_{OCA}$ , and  $T_{OCA'}$  ( $f_{T_{ICA}}$ ,  $f_{T_{OCA}}$ , and  $f_{T_{OCA'}}$ ) are derived by using parameters in the proposed geometric model.

## V. PDFS FOR CHANNEL HOLDING TIMES USING PROPOSED GEOMETRIC MODEL

The formula of pdf from circular cell model of HHO in [8],  $Da_{eq}$  and  $Dtd$  in Fig. 2, and [9] are used to derive the pdf of ICA,  $f_{T_{ICA}}$ , as expressed in equation (9)

$$f_{T_{ICA}}(t) = \begin{cases} \int_{Dtd-Da_{eq}/t}^{V_m} \frac{f_1(w)}{Da_{eq}^2} \frac{1}{V_m} dw, \\ \text{for } \frac{(Dtd - Da_{eq})}{V_m} \leq t \leq \frac{(Dtd + Da_{eq})}{V_m} \\ \\ \int_{Dtd-Da_{eq}/t}^{(Dtd+Da_{eq})/t} \frac{f_1(w)}{Da_{eq}^2} \frac{1}{V_m} dw, \\ \text{for } \frac{(Dtd + Da_{eq})}{V_m} \leq t \end{cases} \quad (9)$$

where

$$f_1(w) = \frac{w\sqrt{-(Dtd^2 - Da_{eq}^2) + 2Da_{eq}(tw)^2 + 2Dtd^2(tw)^2 - (tw)^4}}{\pi w} \quad (10)$$

and  $V_m$  is the maximum velocity of MS.

$Da_{eq}$  and  $Dtd$  of the geometric model are applied in the improved formula of pdf from [8] for finding the pdf of OCA,  $f_{T_{OCA}}$ , as expressed in equation (11)

$$f_{T_{OCA}}(t) = \begin{cases} \int_0^{V_m} \frac{f_2(w)}{(Dtd)^2 - (Da_{eq})^2} \frac{1}{V_m} dw, \\ \text{for } 0 \leq t \leq \frac{(Dtd - Da_{eq})}{V_m} \\ \\ \int_0^{V_m} \frac{f_2(w)}{(Dtd)^2 - (Da_{eq})^2} \frac{1}{V_m} dw - \int_{Dtd-Da_{eq}/t}^{V_m} \frac{f_1(w)}{(Dtd)^2 - (Da_{eq})^2} \frac{1}{V_m} dw, \\ \text{for } \frac{(Dtd - Da_{eq})}{V_m} \leq t \leq \frac{(Dtd + Da_{eq})}{V_m} \\ \\ \int_0^{V_m} \frac{f_2(w)}{(Dtd)^2 - (Da_{eq})^2} \frac{1}{V_m} dw - \int_{Dtd-Da_{eq}/t}^{(Dtd+Da_{eq})/t} \frac{f_1(w)}{(Dtd)^2 - (Da_{eq})^2} \frac{1}{V_m} dw, \\ \text{for } \frac{(Dtd - Da_{eq})}{V_m} \leq t \leq \frac{(Dtd + Da_{eq})}{V_m} \\ \\ \int_0^{2Dtd/t} \frac{f_2(w)}{(Dtd)^2 - (Da)^2} \frac{1}{V_m} dw - \int_{Dtd-Da_{eq}/t}^{(Dtd+Da_{eq})/t} \frac{f_1(w)}{(Dtd)^2 - (Da_{eq})^2} \frac{1}{V_m} dw, \\ \text{for } \frac{2Dtd}{V_m} \leq t \end{cases} \quad (11)$$

where

$$f_2(w) = \frac{w\sqrt{4(Dtd)^2 - (tw)^2}}{\pi} \quad (12)$$

In order to get  $f_{T_{OCA}'}$ , the velocity ( $V$ ) and direction of MS are concerned. The pdf of  $V$  ( $f_v(V)$ ) has been improved from uniform pdf [4] to the pdf depending on MS's velocity [5], [6] as expressed in equation (13)

$$f_v(V) = \frac{2V}{V_m^2}, \quad \text{for } 0 \leq V \leq V_m \quad (13)$$

$\theta$  is the direction of MS moving to the new cell referred to the direction of MS pointing to the center of new cell. The pdf of  $\theta$  ( $f_\theta(\theta)$ ) has been developed from uniform pdf [4] to pdf depending on  $\theta$  [5], [6] as expressed in equation (14)

$$f_\theta(\theta) = \frac{1}{2} \cos \theta, \quad \text{for } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \quad (14)$$

The  $f_\theta(\theta)$  is assumed to be zero for any angle out of range ( $-\pi/2$  to  $\pi/2$ ) and gives very low value at the angle closed to  $-\pi/2$  and  $\pi/2$ . Thus, from Fig. 3, handoff area for an incoming MS can be shaded. For simplicity, the proposed radius of circle for shaded area can be approximated as  $Dh$  expressed in equation (15).

$$Dh = \frac{(Dtd + \sqrt{3}D - Da_{eq})}{2} \quad (15)$$

The pdf of  $OCA'$ ,  $f_{T_{OCA}'}$ , using proposed  $Dh$  from equation (15), formula of pdf for HHO in [8] and the characteristics of  $f_v(V)$  and  $f_\theta(\theta)$  from equations (13) and (14), respectively. The results of  $f_{T_{OCA}'}$  can be expressed as

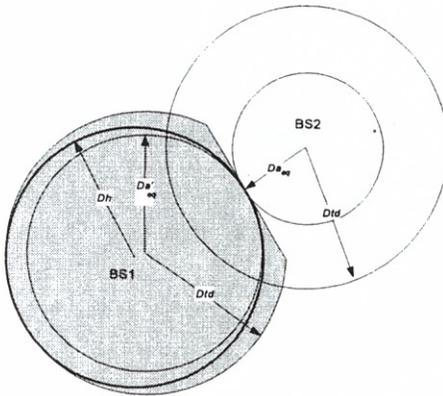


Fig. 3 Handoff area (shaded) and approximated circle with radius  $Dh$ .

$$f_{T_{OCA'}}(t) = \begin{cases} \frac{16(Dh)^2}{3V_m^2t^3} - \frac{8(Dh)^2 + V_m^2t^2}{3(Dh)V_m t^2} \sqrt{\left(\frac{2Dh}{V_m t}\right)^2} - 1, \\ \text{for } 0 \leq t \leq \frac{2(Dh)}{V_m} \\ \frac{16(Dh)}{3V_m^2t^3}, \\ \text{for } \frac{2(Dh)}{V_m} \leq t. \end{cases} \quad (16)$$

Now, all parameters in equation (5) can be found except  $\lambda_{OCA'}$  and  $P_B$ .

## VI. PERFORMANCE INDICATORS AND NUMERICAL RESULTS

One of the important performance indicators in cellular system is PB. Besides PB, there are other indicators: quality of service for communication, trunk-resource efficiency, outage probability, handoff number, carried traffic, and etc [1], [2], [3], [7].

This paper presents the pdfs for channel holding times in the SHO system as shown in equation (6) format. Therefore load balance equation in equation (5), which involves various pdfs, is selected to obtain the important indicator 'PB'. Note that some indicators described above can be obtained by the proposed geometric model but they are not shown in this paper.

The Erlang-B formula [3], [10] is applied to SHO system as shown in equation (17) in order to solve  $P_B$ .  $N$  is the number of users in each cell as well as  $N_{\max}$  is the number of Walsh codes for CDMA system. Notice that  $\lambda_{OCA'}$  and PB are appeared in equation (17).

$$P_B = \frac{(\lambda_{ICA} + \lambda_{OCA} + \lambda_{OCA'})^{N_{\max}}}{\mu^{N_{\max}} (N_{\max})!} P_0 \quad (17)$$

where

$$P_0 = \frac{1}{\sum_{i=0}^{N_{\max}} \frac{(\lambda_{ICA} + \lambda_{OCA} + \lambda_{OCA'})^i}{\mu^i i!}} \quad (18)$$

That is, the two equations, equations (5) and (17), and the two unknown variables,  $\lambda_{OCA}$  and  $P_B$ . Then  $P_B$  can be solved. Next, the examples of numerical results are presented.

Actually,  $D_d$  is related to  $T_{DROP}$  [3]. In this paper,  $T_{DROP}$  is set in range of  $-17$  to  $-13$  dB while  $T_{ADD}$  is set as  $-10$  dB. These SHO thresholds ( $T_{ADD}$ ,  $T_{DROP}$ ) are used for the pilot signal ( $E_c/I_o$ ) in cellular CDMA system as described in section II [3] were computed in order to get  $D_d$  as shown in Fig. 4.

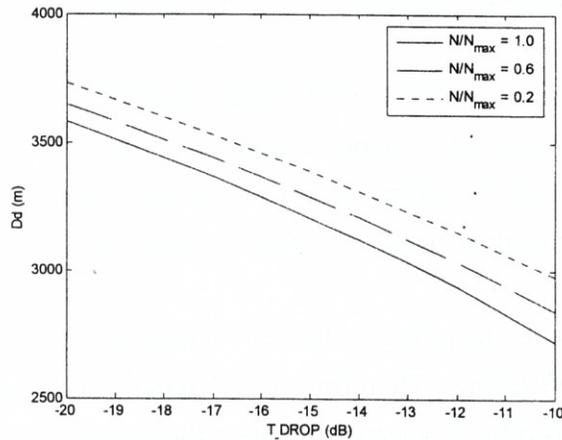


Fig. 4.  $D_d$  as a function of  $T_{DROP}$ .

In Fig. 4,  $D_d$  is decreased when  $T_{DROP}$  is increased. In addition, at high traffic load ( $N/N_{max} = 1$ ),  $D_d$  is less than those at lower traffic load due to the effect of interference from the number of users.

The parameters of cellular CDMA system include BS power 5 watts, pilot signal percentage 15%, overhead percentage 28.5%, voice activity factor 0.4, path loss exponent 4,  $D$  in the proposed model 3,000 m,  $T$  120 ms, and  $N_{max}$  50, and  $V_m$  70 km/hr.

Fig. 5 shows the percentage of  $n$ -way handoff at  $T_{DROP} -15$  dB. Normally, at any  $T_{DROP}$ , there are different percentages of  $n$ -way handoffs. And, these  $n$ -way handoffs (related to number of traffic channels) and  $D_d$  (related to capacity coverage) as shown in Fig. 4 affect the  $P_B$  (related to available channels) of the system.

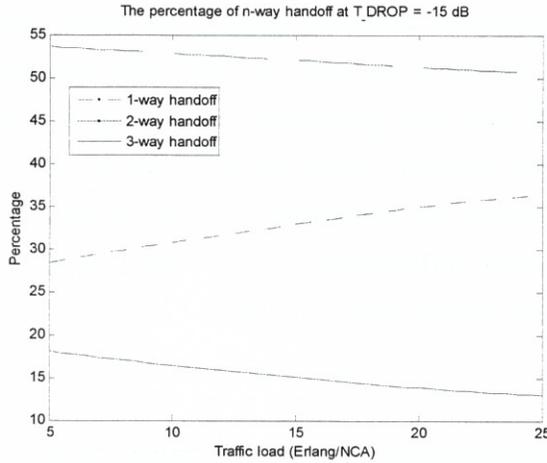


Fig. 5 Dd as a function of T\_DROP.

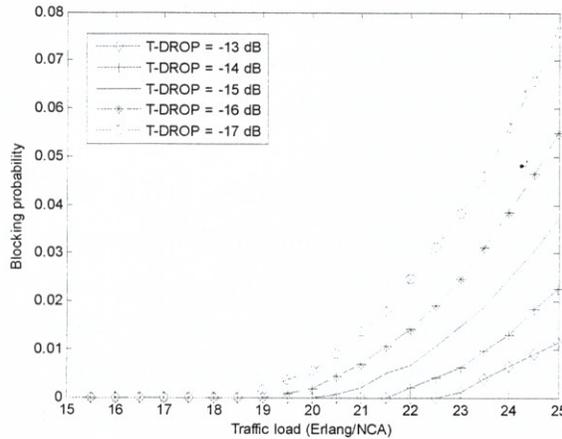


Fig. 6. The blocking probability (PB) as a function of traffic load.

In Fig. 6, the higher T\_DROP can give the lower PB to the system at any traffic load because at the higher T\_DROP, Dd (Fig. 4) of considered cell is lower. Therefore, the 2-w handoff and 3-w handoff can be reduced. This means that the lower coverage can be used to reduce PB in the cellular CDMA system.

### VII. CONCLUSIONS

Under the cellular CDMA system, the pdfs of dwell times in three proposed areas: ICA, OCA, OCA' can be derived in order to obtain the expectation of channel holding times of those three areas. In addition, this paper also shows the method to

get PB by using the derived pdfs according to SHO thresholds, load balance equation, and Erlang-B formula. Besides PB, these resulted pdfs (equations (9), (11), and (16)) accompanied with proposed geometric model (Figs. 2 and 3) can be used to get other indicators for evaluation of the cellular CDMA system.

One way to improve the performance is that the new algorithm applied to the system should consider and develop the percentage of n-way handoff (as the example in Fig. 5) which affects other indicators.

#### REFERENCES

- [1] A. J. Viterbi, A. M. Viterbi, K. S. Gilhousen, and E. Zchavi, "Soft handoff extends CDMA cell coverage and increases reverse link capacity," *IEEE Journal on Selected Areas in Communications*, vol. 12, pp. 1281-1288, 1994.
- [2] D. Wong and T. J. Lim, "Soft handoff in CDMA mobile systems," *IEEE Personal Communications*, pp. 6-17, 1997.
- [3] B. Homnan, W. Benjapolakul, K. Tsukamoto, and S. Komaki, "The effects of varying soft handoff thresholds in cellular CDMA system," *IEICE Transaction on Communications*, vol. E87-B, no. 4, pp. 807-815, 2004.
- [4] D. Hong and S. S. Rappaport, "Traffic model and performance analysis for cellular mobile radio telephone system with prioritized and nonprioritized handoff procedures," *IEEE Transactions on Vehicular Technology*, vol. 35, no. 3, pp. 77-92, 1986.
- [5] H. Xie and D. Goodman, "Mobility models and biased sampling problem," *IEEE ICUPC*, pp. 803 - 807, 1993.
- [6] S. Phokeewanichkul, K. Chamnanya, and B. Homnan, "An analysis of mobility model in cellular communication system using Gaussian probability density function for handover direction," *IASTED CSS*, pp. 378-383, 2003.
- [7] B. Homnan, V. Kunsriruksakul, and W. Benjapolakul, "A comparative performance evaluation of soft handoff between IS-95A and IS-95B/cdma2000," *IEEE APCCAS*, pp. 34-37, 2000.

- [8] D. -W. Tcha, S. -Y. Kang, and G. -W. Jin, "Load analysis of the soft handoff scheme in CDMA cellular system," IEEE Journal on Selected Areas in Communications, vol. 19, no. 6, pp. 1147 - 1152.
- [9] D. K. Kim and D. K. Sung, "Characteristics of soft handoff in CDMA systems," IEEE Transactions on Vehicular Technology, vol. 48, no. 4, 1999.
- [10] L. Kleinrock, Queueing systems volume I: theory, John Wiley & Sons, 1890.