Thammasat Economic Journal
P-ISSN : 2630-0931
E-ISSN : 2651-0529
Homepage : *https://www.tci-thaijo.org/index.php/TER*
Vol. 32, No. 1, January-April 2014 Page [54-70]

Imperfect Consumer Information and Firm Agglomeration

*Thanin Wangveerathananon**
Faculty of Economics, Thammasat University, Thailand

Abstract

Conventional microeconomics concludes that firms prefer high demand but low competition. However, in many locations where firms sell a homogeneous product agglomerates are evidenced. When consumers have imperfect information about selling locations and location search is prohibited, each location is identical so they choose a location to visit randomly. When the number of locations increases, the expected demand in each location decreases, creating demand uncertainty in each location. The existence of an active store in a particular location guarantees that it has sufficient demand to sustain business. Anew firm selling a similar product must consider the tradeoff between choosing a location with certain demand but high competition or locations with uncertain demand but possible low competition. This tradeoff is the main study of this paper. If the number of locations exceeds the threshold level, all firms are willing to agglomerate in the location with a certain demand. Otherwise, a location with uncertain demand can coexist with agglomerated location.

Keywords: Agglomeration, Imperfect information, Price dispersion, Spatial economics, Non-cooperative game
JEL Classifications: C72, D43, L10, L13, R12, R32

* **Address:** 2 Prachan Road, Pranakorn, Bangkok Thailand, 10200. Email: thaninwng@msme.au.edu

Imperfect Consumer Information and Firm Agglomeration

*ธนินท์ วังวีระธนานนท์**

คณะเศรษฐศาสตร์ มหาวิทยาลัยธรรมศาสตร์ ประเทศไทย

บทคัดย่อ

โดยทั่วไปทฤษฎีเศรษฐศาสตร์จุลภาคสรุปว่าหน่วยธุรกิจชอบอุปสงค์ที่สูงแต่มีการแข่งขันต่ำ อย่างไรก็ดีมีหลักฐานเป็นที่ประจักษ์ถึงหลายสถานที่ซึ่งร้านค้าที่ขายสินค้าชนิดเดียวกันมาอยู่รวมกันยัง เมื่อผู้บริโภคมีความไม่สมบูรณ์ของข้อมูลในเรื่องของสถานที่ขายสินค้าและการค้นหาสถานที่ถูกจำกัด สถานที่แต่ละแห่งจึงไม่มีความแตกต่างกันทำให้ผู้บริโภคทำการสุ่มสถานที่ขายที่จะเดินทางไปแบบเท่าเทียม เมื่อจำนวนสถานที่มีมากขึ้นอุปสงค์เฉลี่ยของแต่ละสถานที่จึงลดลงซึ่งทำให้เกิดความไม่แน่นอนของอุปสงค์ในแต่ละสถานที่ การมีอยู่ของร้านค้าซึ่งดำเนินการอยู่ในสถานที่ใดที่หนึ่งเป็นเครื่องยืนยันว่าสถานที่นั้นมีอุปสงค์ที่เพียงพอ หน่วยธุรกิจที่ขายสินค้าชนิดเดียวกันจะต้องพิจารณาข้อดีข้อเสียที่แลกกันระหว่างสถานที่ที่มีความแน่นอนของอุปสงค์แต่มีการแข่งขันสูงหรือสถานที่ที่มีความไม่แน่นอนของอุปสงค์แต่เป็นไปได้ที่จะมีการแข่งขันต่ำ ข้อดีข้อเสียที่แลกกันนี้เป็นการศึกษาหลักของบทความนี้ ถ้าจำนวนสถานที่มีมากขึ้นจนเกินค่าขีดกั้น หน่วยธุรกิจทั้งหมดจะรวมตัวอยู่ในที่เดียวกัน หรือมิฉะนั้นสถานที่ที่มีความไม่แน่นอนของอุปสงค์สามารถอยู่ร่วมกับสถานที่ที่มีการรวมตัวของหน่วยธุรกิจ

1. Introduction

In theory, firms prefer high demand but low competition. In reality, we found many locations where firms selling a homogeneous product are concentrated. So far, explanation of the contradiction between theoretical postulation and empirical evidence has been offered by spatial economics, pioneered by Hotelling (1929), and the new economic geography of Krugman (1991). Their main explanation points to firm location decisions depending on transportation cost and distance between them and consumers. This would operate under the implicit assumption that information about firm location is assumed to be known by all agents. Hence, firms tend to minimize transportation costs by being located close to consumers so that firm agglomeration can be observed.

In the real world, assuming perfect information of firm location seems counter-intuitive. Consumers have imperfect information on prices and selling location. As echoed in search theory literature selling and location by Stigler (1961), information friction regarding prices is the main reason for persistence of price dispersion[1]. In this research a lack of knowledge of prices is the main reason for search not the knowledge about the selling location. Firm location is also assumed to be known by all agents.

Interestingly, these two branches of literature have so far been treated mutually exclusive. This paper is the first attempt to study firm location decisions when consumers have imperfect information on prices and the selling location of a particular product. When consumers do not have prior information about selling location, they choose a location to visit randomly. When the number of potential locations rises, the expected demand in each location falls due to uniform randomization. This is the uncertainty facing a firm which chooses a location with consumer imperfect information in the selling location.

Against this backdrop, the objective of this paper is to integrate location into the price dispersion model, using a non-sequential search. When combining locations into the price dispersion model, imperfect information about the selling prices of firms in each location keeps prices in each location within a range between marginal cost and reservation price. Thus, competition in each location is not perfect and firms in the same location can achieve positive normal profit.

There are three contributions of this paper to the existing literature: first, the model in this paper applies non-cooperative games to analyze location choices of firms, using the model developed in Burdett and Judd (1983) as a point of departure. In this paper, consumers do not have perfect information regarding firm location and prices offered by each, in which they are capable of price search but not location search; secondly, while Takahashi (2013) incorporates an imperfect information aspect in a new economic geography model, the information emphasizes product variety instead of firm location. So, transportation cost still plays a significant role; third, the model in this paper can be used as a workhorse for future research to add location search into search theory literature. When the consumer location search is enabled, the result is that consumer location decision is endogenized, given the firm location decision[2].

---

[1] See the survey by Baye, Morgan, and Scholten (2006)

[2] Subsequent work by the author endogenized consumer location decision by introducing two-step searching, which is a search method that doubles the non-sequential search process. The result is that consumer location distribution depends on firm location distribution. Its main contribution is that firm agglomeration leads to consumer agglomeration, since it induces consumer location search that stimulates the popularity of the agglomerated location.

2. Model

Consider a world represented by a circle where M identical uninformed consumers live in the center and N identical firms choose to locate in locations around the circumference. There are K identical locations and one pre-existing market, denoted by location 0, in the location set, $\bar{K} = \{0,1,2,...,k,...,K\}$, $K \in \mathbb{N}$. The terms locations and markets are interchangeable in this paper to represent only potential transactable locations, not just any spaces. The superscript of a variable denotes the location it belongs to. For example, N^k is the number of firms at location k. In the pre-existing market, there are I_F^0 pre-existing (incumbent) firms already established and I_C^0 pre-existing consumers aware of the existence of this market. So there are a total of $K+1$ locations, $N+I_F^0$ firms, and $M+I_C^0$ consumers. Note that only location 0 in this model has pre-existing firms and consumers, or $I_F^k, I_C^k = 0, \forall k \in \bar{K} \setminus \{0\}$.

The firms choose a location to locate their sole stores in the location game and choose price distribution in the price game, given that they can supply indefinitely at a constant marginal cost r and the competition in each location is given as in Burdett and Judd (1983). Consumers have an inelastic demand of one unit of homogeneous product with valuation v for which they must travel to one of the locations to find the firms. Therefore, expected demand in each location is equal to the number of consumers in each location. Consumers know where all the locations are, but not where all firms are located. Once they arrive at a location, they know the price distribution but do not know exactly which firms offer which prices.

When consumers have imperfect information about selling locations and location search is prohibited, each location is identical to them, so they choose for a location to visit randomly. Therefore the expected number of them in each location is the same, or $E(M^k) = \frac{M}{K+1}, \forall k \in \bar{K}$.[3] The expected demand in each location falls as the number of potential locations K rises. This is the demand uncertainty created by uniform randomization of the imperfect information consumers. However, since only location 0 has a certain demand from pre-existing consumers, the probability that a firm will locate in the pre-existing market, which represent the degree of agglomeration, rises as expected demand in other locations falls. In the symmetric case, if there are no pre-existing firms and consumers in location 0, then all $K+1$ locations are identical. Therefore the probability that a firm will locate in one of K locations is $1/(K+1)$ and hence the number of firms in each location is $N/(K+1)$

The analysis of the model is divided into three types of games: search game, price game, and location game, for a total of $2(K+1)+1$ games. The sequence of games is summarized as follows: The location game is a two-stage simultaneous moves game. In the first stage, firms and consumers choose locations simultaneously. However, since consumers have imperfect information, their location decision is given exogenously by uniform randomization. Therefore only firms choose a location, given that the competition in each location is determined by the search game and the price game. In the second stage, firms that choose a particular location and consumers who arrive there play the price game and search game at that location simultaneously with only their kind, given the result of

[3] Without loss of generality, assume that parameters M and K are such that , $\frac{M}{K+1} \in \mathbb{N}$,

the other game. The equilibrium of both games form the general equilibrium, or the market equilibrium. Firms and consumers take into account what the others would do in the equilibrium. Each price game and search game combined are virtually the price dispersion model of Burdett and Judd (1983) with slight modifications to precisely express the number of firms. The location game is played at the root node and contains *2(K+1)* subgames which are *K+1* search games and *K+1* price games played in the second stage at each *K+1* node (location).

The location game is played first, but discussed last due to backward induction, by firms which compete with each other in choosing location distribution to maximize their profit, given that competition in each location will be according to Burdett and Judd (1983). In the location game, each firm has the same number of strategies, which is the number of locations. Consequently, the number of locations is also the number of price games (as well as search games), whose number of players are determined by the location game.

In search games, consumers play by choosing the number of quotations to sample, with cost *c* per sample, from firms in that location, given the price distribution. The consumer strategy is called the search behavior. Since travelling to more than one location is restricted, they purchase the product from the firm offering the lowest price in their samples, if it is less than *v*. Otherwise they will not participate.

In price games, firms compete by choosing price distribution to maximize profit, given consumer search behavior. Firms are said to play a pure strategy if they choose to charge the consumers only one price, or the price distribution is degenerate. Price dispersion is a situation where firms randomize prices according to some at omless distributions. The price distributions of firms are mixed strategies. Since all are identical, so is the equilibrium price distribution. Pre-existing firms and consumers are passive players in the sense that they do not make location decisions, but still must choose a price to offer or a number of quotations to sample, for which they have the same production and search cost.

3. Search Game

A search game $G_S^k = \left\langle \bar{M}^k, \{S_m\}_{m=1}^{M^k}, \left\{u_m\left(s_m : \bar{F}^k\right)\right\}_{m=1}^{M^k} \right\rangle$ at location *k* is a simultaneous move game played by *E(M^k)* consumers with the strategy set S_m and utility function $u_m\left(s_m : \bar{F}^k\right)$ for each consumer $m \in \bar{M}^k$, given the price distribution $\bar{F}^k$ of the price game at location *k* The set of players in this game is $\bar{M}^k \equiv \left\{1, 2, ..., m, ..., E\left(M^k\right) + I_C^k\right\}, \ \forall k \in \bar{K},$ which is the set of consumers who arrive at location *k* An arbitrary player, but not *m*, is referred to as $m' \in \bar{M}^k \setminus \{m\}$.

The information structure of the games is as follows: within a location, consumers know the location of each shop, but do not know the price offered by each of them so they must pay some cost to acquire the quotation. Consumers are identical in terms of information and search cost and these characters are common knowledge in the game.

3.1 Strategy

Consumer strategy sets are the number of firms they would like to sample to observe prices. At location *k*, consumers can sample either one, two, or up to $N^k + I_F^k$ firms. Let s_m denotes the strategy of consumer $m \in \bar{M}^k$, then $s_m \in \left\{1, 2, ..., N^k\right\} \equiv \hat{S}_m$ where $\hat{S}_m$ is consumer *m*'s strategy set. However, it will be shown in the equilibrium that sampling more than two firms is never an optimum strategy for any consumers. Therefore the consumer strategy sets are reduced from $\hat{S}_m$ to $\bar{S}_m = \{1, 2\}$. However, if the consumers are allowed to sample more than two prices – for example, three – they will never be willing to do so. This results from the fact that when all consumers sample two firms, the equilibrium outcome is Betrand competition, see Burdett and Judd (1983) and Baye, Morgan, and Scholten (2006). The set of all possible pure strategy profiles in the game is $S^k \equiv \times_{m\in\bar{M}^k} \bar{S}_m$, while the set of pure strategy profiles of all players other than *m* is $S^k_{-m} \equiv \times_{m'\in\bar{M}^k\setminus\{m\}} \bar{S}_{m'}$.

Consumer ‘s mixed strategy is a probability mass function $f_{\eta_m}(s_m)$ of a random variable $\eta_m : S^k_{-m} \to \bar{S}_m$ that takes values from its pure strategy set, $\bar{S}_m$, and defined by $f_{\eta_m}(s_m) \equiv \mu(\eta_m \in \{s_m\})$, $s_m \in \bar{S}_m$ and $\sum_{s_m\in S_m} f_{\eta_m}(s_m) = 1$.[4] Since for each $m \in \bar{M}^k$, $\bar{S}_m$ has only two elements, the mixed strategy of consumer *m* can also be defined by $q_m = f_{\eta_m}(1)$ and $1-q_m = f_{\eta_m}(2)$, $q_m \in [0,1]$. That is, the mixed strategy of consumer *m* can be represented by the probability q_m, that he will sample one firm. The set of mixed strategy of consumer *m* is the interval $[0,1] \equiv Q_m$. At location *k*, the set of all mixed strategy profiles of the game is $Q^k \equiv \times_{m\in\bar{M}^k} Q_m$,while the set of mixed strategy profiles of all players other that *m* is $Q^k_{-m} \equiv \times_{m'\in\bar{M}^k\setminus\{m\}} Q_{m'}$. Let $\bar{q}^k \in Q^k$ and $\bar{q}^k_{-m} \in Q^k_{-m}$, then $\bar{q}^k = (q_1, q_2, ..., q_m, ..., q_{M^k})$, $\exists q_m \in Q_m$, $\forall m \in \bar{M}^k$, and $\bar{q}^k_{-m} = (q_1, q_2, ..., q_{m-1}, q_{m-1}, ..., q_{M^k})$, $\exists q_{m'} \in Q_{m'}$, $\forall m' \in M^k \setminus \{m\}$.

3.2) Payoff

Since all consumers other than *m* are identical, they play the same mixed strategy. The mixed strategy profile of all the consumers other than *m* when they play the same mixed strategy *q* is denoted by $\bar{q}_{-m} \equiv (q)_{m'\in\bar{M}^k\setminus\{m\}} \in Q^k_{-m}$. The mixed strategy profile that all the consumers play the same mixed strategy *q* is also denoted in the same way by $\bar{q} \equiv (q)_{m\in\bar{M}^k} \in Q^k$. Price distribution at location *k* is the firms’ mixed strategy profile $\bar{F}^k = (F_1, F_2, ..., F_n, ..., F_{N^k})$, $\exists F_n \in \Sigma_n$, $\forall n \in \bar{N}^k$. Since all firms are identical, in the Mixed-Strategy Nash Equilibrium (MSNE) of the price game at any locations, all mixed strategies in the MSNE profile $\bar{F}^*$ are the same. That is, whichever firm is sampled, consumer *m* always face the same price distribution $F^*(p:\bar{q}_{-m})$.

Consumer *m*’s expected utility is his valuation of the product *v* minus the expected minimum price of s_m samples, $E(\min(p): s_m, F^*(p:\bar{q}_{-m}))$ and the associated search costs. The search cost per sample is *c*, so if a consumer samples s_m firms, his search cost is simply cs_m Then consumer *m*’s expected utility when he play the pure strategy s_m and believes all other consumers are playing the mixed strategy profile $\bar{q}_{-m}$ and given all firms play the mixed strategy $F^*(p:\bar{q}_{-m})$ is

$$\begin{aligned} u(s_m : \bar{q}_{-m}, F^*(p:\bar{q}_{-m})) &= (v - cs_m) - E(\min p : s_m, F^*(p:\bar{q}_{-m})) \\ &= (v - cs_m) - \int_{p_{\min}}^{v} (1 - F^*(p:\bar{q}_{-m}))^{s_m} dp. \end{aligned} \tag{1}$$

When consumer *m* plays the pure strategy s_m=1, his expected utility from (1) is

$$\begin{aligned} u(1 : \bar{q}_{-m}, F^*(p:\bar{q}_{-m})) &= (v - c) - E(\min p : 1, \bullet) \\ &= (v - c) - \int_{p_{\min}}^{v} (1 - F^*(p:\bar{q}_{-m})) dp. \end{aligned} \tag{2}$$

---

[4] Let $(\bar{S}_{-m}, \tilde{S}_{-m}, \mu)$ be a probability space, $\tilde{S}_{-m}$ is the Borel-algebra satisfying the usual properties, and $\mu : \tilde{S}_{-m} \to [0,1]$ is a probability measure such that $\mu(\bar{S}_{-m}) = 1$.

When he plays the pure strategy s_m=2, his expected utility from (1) is

$$u\left(2:\bar{q}_{-m}, F^*\left(p:\bar{q}_{-m}\right)\right)=\left(v-2c\right)-E\left(\min p:2, \bullet\right)$$
$$=\left(v-2c\right)-\int_{p_{\min}}^{v}\left(1-F^*\left(p:\bar{q}_{-m}\right)\right)^2 dp. \quad (3)$$

Consumer *m*'s best-response correspondence is a point-to-set function that maps a mixed strategy profile at location *k* chosen by all the consumers other than himself to set of his mixed strategies that give him the highest payoff as defined by

$$BR_m\left(\bar{q}_{-m}^k\right)=\left\{q_m^*\in Q_m : u\left(q_m^*:\bar{q}_{-m}^k\right)=\sup_{q_m} u\left(q_m:\bar{q}_{-m}^k\right)\right\},\ \forall \bar{q}_{-m}^k\in Q_{-m}^k. \quad (4)$$

The set of all the mixed strategy BR of the price game at location *k* is defined by $BR\left(\bar{q}^k\right)\equiv \times_{m\in \bar{M}^k} BR_m\left(\bar{q}_{-m}^k\right)$.

3.3 Search Equilibrium

The mixed strategy profile when all consumers other than *m* play the mixed strategy q^* is denoted by $\bar{q}_{-m}^*\equiv\left(q^*\right)_{m'\in\bar{M}^k\setminus\{m\}}$. For firm equilibrium, there are a pure strategy Nash equilibrium (PSNE) and a MSNE depending on the given equilibrium price distribution $F^*\left(p:\bar{q}_{-m}^*\right)$, which is the MSNE of the firms in the price game. A consumer plays a mixed strategy when he is indifferent to playing any pure strategy. The mixed strategy profile $\bar{q}_{-m}^*$ of all consumers other than *m* at location *k* that gives consumer *m* the same expected utility when sampling one firm or two firms can be found by the equal expected utility condition

$$u\left(1:\bar{q}_{-m}, F^*\left(p:\bar{q}_{-m}\right)\right)\equiv u\left(2:\bar{q}_{-m}, F^*\left(p:\bar{q}_{-m}\right)\right)$$
$$c=E\left(\min p:1, \bullet\right)-E\left(\min p:2, \bullet\right)$$
$$=\int_{p_{\min}}^{v}\left(1-F^*\left(p:\bar{q}_{-m}\right)\right)dp-\int_{p_{\min}}^{v}\left(1-F^*\left(p:\bar{q}_{-m}\right)\right)^2 dp\equiv G\left(q\right). \quad (5)$$

When $F^*\left(p:\bar{q}_{-m}\right)$ is degenerate, *p* is a constant random variable and the expected price is a constant. In other words, $E\left(\min p:1, \bullet\right)=E\left(\min p:2, \bullet\right)$. That is $u\left(1:\bar{q}_{-m}, F^*\left(p:\bar{q}_{-m}\right)\right)>u\left(2:\bar{q}_{-m}, F^*\left(p:\bar{q}_{-m}\right)\right)$ and hence consumer *m*'s BR is q_m=1 In the price game, we can see that $F^*\left(p:\bar{q}_{-m}^*\right)$ is degenerate when $q^*\in\{0,1\}$. When $q^*=0$, $F^*\left(p:\bar{q}_{-m}^*\right)$ degenerates at the marginal cost *r*, and when $q^*=1$, $F^*\left(p:\bar{q}_{-m}^*\right)$ at the maximum price *v*. Since consumer *m*'s BR is q_m=1 when $F^*\left(p:\bar{q}_{-m}^*\right)$. is degenerate, then $\bar{q}^*\equiv\left(q^*\right)_{m\in\bar{M}^k}\in BR\left(\bar{q}^*\right)=\times_{m\in\bar{M}^k} BR_m\left(\bar{q}_{-m}^*\right)$, only when $q^*=1$. Thus, $\bar{q}^*$ is a MSNE of the search game when $F^*\left(p:\bar{q}^*\right)$ is degenerate. In other words, when $q^*=0$, a consumer has an incentive to deviate from sampling two prices $\left(q_m=0\right)$ to sampling only one price $\left(q_m=1\right)$, i.e. $\{0\}\notin BR_m\left(\bar{q}_{-m}^*\right)$. On the other hand, when $q^*=1$, a consumer has no incentive to deviate from sampling only one price to other strategy, i.e. $\{1\}\in BR_m\left(\bar{q}_{-m}^*\right)$. Therefore, the MSNE of the search game given that $F^*\left(p:\bar{q}_{-m}^*\right)$ is degenerate only exists when it degenerates at the maximum price.

When $F^*\left(p:\bar{q}_{-m}\right)$ is not degenerate (not constant), $G(q)$ is hump-shaped. The following result is the main finding of Burdett and Judd (1983): there exists a unique $\bar{c}\in\mathbb{R}$ that has three properties. First, there is the unique q^* such that $G_m\left(q^*\right)=\bar{c}$. Second, if $c<\bar{c}$, then there are two q^*, such that $G\left(q_1^*\right)=G\left(q_2^*\right)=c$. Last, if $c>\bar{c}$, then there is no *q* such that $G_m\left(q\right)=c$.

Let $q^* \in (0,1)$ be defined by $G_m(q^*) = c,\ c \le \overline{c}$ where q^* might not be unique. When all consumers other than *m* play the same mixed strategy profile $\overline{q}^*_{-m}$, consumer *m* has the same expected utility whether he sample only one firm or two. Thus any $q_m \in (0,1)$ is consumer *m*'s mixed strategy BR. $(q^*)_{m\in\overline{M}^k} \equiv \overline{q}^* \in BR(\overline{q}^*) = \times_{m\in\overline{M}^k} BR_m(\overline{q}^*_{-m})$,Then where $\overline{q}^*$ is a fixed point of $BR(\overline{q}^*)$. Thus the mixed strategy profile is a MSNE of the search game, where $q^* \in (0,1)$ can have at most two values, depending on the value of the search cost *c*. The MSNE is symmetric and also unique since all consumers are identical which make the game symmetric. The order of player's has no effect on the outcome of the game.

4. Price Game

A price game $G_P^k = \left\langle \overline{N}^k, \{P_n\}_{n=1}^{N^k}, \{\pi(p_n : \overline{q})\}_{n=1}^{N^k} \right\rangle$ at location *k* is a simultaneous move game played by N^k firms with the strategy set P_n and the profit function $\pi_n(p_n : \overline{q})$ for each firm $n \in \overline{N}^k$, given consumer search behavior $\overline{q}$. The set of players in this game is $\overline{N}^k \equiv \{1, 2, ..., n, ..., N^k + I_F^k\},\ N^k \in \mathbb{N}$, which is the set of the firms that choose location *k* as their strategies in the location game, where N^k is their number. Note that $I_F^k, I_C^k = 0, \forall k \in \overline{K} \setminus \{0\}$. An arbitrary player, but not *n*, is referred to as $n' \in \overline{N}^k \setminus \{n\}$.

The information structure of the game is as follows: the price of each firm is private information known only to the firm and consumers who sample them, and cannot be observed by other players in games, since firms set prices simultaneously[5]. Firms know the price distribution of other firms, in that they have some belief about other players' mixed strategies, which is correct in Nash equilibrium.

4.1 Strategy

Each firm has an infinite number of strategies. They can choose the price of their homogeneous product to be any number in the closed interval between constant marginal costs *r* up to the maximum price *v*. In other words, firm *n*'s strategy is the price $p_n \in [r, v] \equiv P_n$ where P_n is firm *n*'s strategy set, identical for all firms in any locations. At location *k*, the set of all possible pure strategy profiles of the price game is $P^k \equiv \times_{n\in\overline{N}^k} P_n$ while the set of pure strategy profiles of all the firms other than *n* is $P_{-n}^k \equiv \times_{n'\in\overline{N}^k\setminus\{n\}} P_{n'}$. Firm *n*'s mixed strategy is a distribution $F_{\sigma_n}(p_n : \overline{q})$ of a random variable $\sigma_n : P_{-n}^k \to P_n$ defined by $F_{\sigma_n}(p_n : \overline{q}) = \mu(\sigma_n \le p_n),\ p_n \in P_n$.[6]

$\Sigma_n = \{F_{\sigma_n} : F_{\sigma_n}(p_n : \overline{q}) \equiv \mu(\sigma_n \le p_n), p_n \in P_n\}$ is the set of all mixed strategies of firm *n*. When the context is clear F_{σ_n} is abbreviated as F_n. At location *k*, the set of all the mixed strategy profile is $\Sigma^k \equiv \times_{n\in\overline{N}^k} \Sigma_n$ while the set of the mixed strategy profile of all firms other than *n* is $\Sigma_{-n}^k \equiv \times_{n'\in\overline{N}^k\setminus\{n\}} \Sigma_{n'}$. Let $\overline{F}^k \in \Sigma^k$ then $\overline{F}^k = (F_1, F_2, ..., F_n, ..., F_{N^k}),\ \exists F_n \in \Sigma_n,\ \forall n \in \overline{N}^k$ and let $\overline{F}_{-n}^k \in \Sigma_{-n}^k$ then $\overline{F}_{-n}^k = (F_1, F_2, ..., F_{n-1}, F_{n+1}, ..., F_{N^k}),\ \exists F_{n'} \in \Sigma_{n'},\ \forall n' \in \overline{N}^k \setminus \{n\}$.

4.2) Payoff

At location *k*, given the consumers mixed strategy profile $\overline{q}$, the expected number of consumers who sample only one firm and two firms are $qE(M^k)$ and $(1-q)E(M^k)$, respectively. The maximum price a firm can charge equals the consumer identical product valuation *v*, while the price consumers expect to engage in price searching is

[5] When the firm knows the price that the other firm is charging or when all consumers sample at least two firms, the equilibrium collapses to Bertrand competition.

[6] Let $(\overline{P}_{-n}, \tilde{P}_{-n}, \mu)$ be a probability spaces, $\tilde{P}_{-n}$ is a Borel-algebras satisfying the usual properties, and $\mu : \tilde{P}_{-n} \to [0,1]$ is a probability measures such that $\mu(\overline{P}_{-n}) = 1$.

$E(p) = E(\min p : 1, \bullet) \le v - c$. The probability that a firm in location k will be sample by a consumer is $1/N^k$.

Since all firms are identical, they play the same mixed strategy, or $F_{n'} = F'(p_{n'} : \bar{q}),\ \forall n' \in \bar{N}^k \setminus \{n\}$, given the consumer mixed strategy profile $\bar{q}$. The mixed strategy profile of all the firms other than n when they play the same mixed strategy $F'(p_{n'} : \bar{q})$ is $\bar{F}'_{-n} \equiv \left(F'(p_{n'} : \bar{q})\right)_{n' \in \bar{N}^k \setminus \{n\}}$.When all firms other than n play the mixed strategy profile $\bar{F}'_{-n}$, whenever a consumer samples two firms, both play the mixed strategy $F'(p_{n'} : \bar{q})$. Then, given the consumer mixed strategy profile $\bar{q}$, firm n's profit function when it play the pure strategy p_n at location k is

$$\pi_n^k\left(p_n : \bar{q}, \bar{F}'_{-n}\right) = \begin{cases} (p_n - r) E\left(M^k\right)\left[q \dfrac{1}{N^k} + (1-q)\dfrac{2}{N^k}\left(1 - F'(p_n : \bar{q})\right)\right], & \text{if } p_n \le v \\ 0, & \text{if } p_n > v, \end{cases} \tag{6}$$

where p_n- r is the unit margin and $1 - F'(p_n : \bar{q})$ is the probability that p_n will be lower than the price of the other sampled firm.

Firm n's best response correspondence (BR) is a point-to-set function that map a mixed strategy profile of all firms other than n to the set of optimal mixed strategies, which is

$$BR_n\left(\bar{F}_{-n}^k\right) = \left\{F_n^* \in \Sigma_n : \pi_n\left(F_n^* : \bar{F}_{-n}^k\right) = \sup_{F_n}\left(F_n : \bar{F}_{-n}^k\right)\right\},\ \forall \bar{F}_{-n}^k \in \Sigma_{-n}^k. \tag{7}$$

The set of all the mixed strategy BR of the price game at location k is defined by $BR\left(\bar{F}^k\right) = \times_{n \in \bar{N}^k} BR_n\left(\bar{F}_{-n}^k\right)$.

4.3 Firm Equilibrium and Market Equilibrium

For the firm equilibrium, there are two PSNEs and one MSNE depending on the given search behavior q^* of all consumers. The MSNE is symmetric and also unique since all firms are identical, which make the game symmetric. The general equilibrium or market equilibrium is a pair of strategy profiles (pure or mixed) of the search game and the price game that satisfy the equilibrium for both games.

When $q^* = 0$ (all consumers sample two prices), if firm n charges p_n lower than firm n' charges $p_{n'}$, it wins the consumers who sample it. When they charge the same price, each is assumed to share half the demand. Whichever counterparty firm is sampled by consumers, firm n always tries to undercut it. Thus the best-response given to any mixed strategy profile is the pure strategy $p_n = r$. This is called the competitive (Bertrand) equilibrium. Competitive equilibrium does not exist in general equilibrium, since it does not exist in the search equilibrium as consumers deviate from sampling two prices ($q_m = 0$) to sampling only one price ($q_m = 1$).

When $q^* = 1$ (all consumers sample only one price), firm n's profit does not depend on what other firms might play and simply charge the maximum price possible. Thus the best-response given to any mixed strategy profile is the pure strategy $p_n = v$. This is the monopoly equilibrium, which exists in the general equilibrium since it exists in the search equilibrium. Note that consumers do not deviate from sampling only one price ($q_m = 1$). The monopoly equilibrium is also called Diamond's paradox after Diamond (1971).

When $q^* \in (0,1)$, firm n plays a mixed strategy if any pure strategies give the same expected profit. The mixed strategy profile $\bar{F}_{-n}^* \equiv \left(F^*(p_{n'} : \bar{q})\right)_{n' \in \bar{N}^k \setminus \{n\}}$ of all firms other than n at location k that make firm n's in different to playing pure strategies can be found by the equal expected profit condition

$$\pi_n^k\left(v:\bar{q}',\bar{F}_{-n}^*\right)=\left(v-r\right)\frac{E\left(M^k\right)}{N^k}q$$
$$\equiv\left(p_n-r\right)\frac{E\left(M^k\right)}{N^k}\left[q+2\left(1-q\right)\left(1-F^*\left(p_n:\bar{q}\right)\right)\right]=\pi_n^k\left(p_n:\bar{q},\bar{F}_{-n}^*\right). \tag{8}$$

Solving for $F^*\left(p_n:\bar{q}\right)$ yields

$$F^*\left(p_n:\bar{q}\right)=\begin{cases}1, & \text{if } p_n\geq v\\ 1-\left(\dfrac{v-p_n}{p_n-r}\right)\left(\dfrac{q}{2\left(1-q\right)}\right), & \text{if } p_{\min}<p_n<v\\ 0, & \text{if } p_n\leq p_{\min},\end{cases} \tag{9}$$

where

$$p_{\min}=r+\left(v-r\right)\frac{q}{2-q} \tag{10}$$

can be found by solving for p_n in (8)given $F^*\left(p_n:\bar{q}\right)=0$. When substituting (9) into (6), firm n's profit function does not depend on its strategy (pure or mixed).Therefore, when all the firms other than n play the mixed strategy profile $\bar{F}_{-n}^*$, $BR_n\left(\bar{F}_{-n}^*\right)=\Sigma_n$. That is any strategies in firm m's mixed strategy set is the BR to the mixed strategy profile $\bar{F}_{-n}^*$ of all firms other than n. The set of the mixed strategy BR of the price game at location k given the mixed strategy profile $\bar{F}^*$ is $BR\left(\bar{F}^*\right)=\times_{n\in\bar{N}^k}BR_n\left(\bar{F}_{-n}^*\right)=\Sigma^k$, which is also the set of all mixed strategy profiles of the game. Thus $\bar{F}^*\in BR\left(\bar{F}^*\right)$, or $\bar{F}^*$ is a fixed point of $BR\left(\bar{F}^*\right)$, and the mixed strategy profile $\bar{F}^*$ is the MSNE of the price game at location k. The mixed strategy profile $\bar{q}^*$ of the search game along with the mixed strategy profile $\bar{F}^*\left(p:\bar{q}^*\right)$ of the price game form the market equilibrium at location k.

5. Location Game

A location game $G_L=\left\langle\bar{N},\left\{\bar{K}\right\}_{n=1}^N,\left\{\pi_n\left(l_n\right)\right\}_{n=1}^N\right\rangle$ is a simultaneous move game played by N firms with the profit function $\pi_n\left(l_n\right)$ of each firm $n\in\bar{N}$ and the strategy set $\bar{K}=\left\{0,1,2,...,k,...,K\right\}$, $K\in\mathbb{N}$, given consumers' $\bar{q}^*$ and the firms' $\bar{F}^*\left(p:\bar{q}^*\right)$ mixed strategy profile of the search game and price game at each location k, respectively. The set of players is $\bar{N}\equiv\left\{1,2,...,n,...,N\right\}$, $N\in\mathbb{N}$, where N is the number of firms and $\sum_{k=0}^{K}N^k=N$. An arbitrary player, but not n, is referred to as $n'\in N\setminus\left\{n\right\}$.

The information structure of the game is as follows: since the location game is played simultaneously among firms, no firm is able to observe any firm or consumer movement before it makes a decision. That is a firm makes decisions based on strategies it believes the other firms will play. The firms know that consumers do not know the number of firms in each location and obtain random uniformity for a location to visit.

5.1 Strategy

The pure strategy set of a firm is the location set $\bar{K}$ and all firms have the same strategy set. Firm n's pure strategy, denoted by $l_n\in\bar{K}$, is the location chosen to setup its store. The set of all the possible pure strategy profiles in the game is $\bar{K}_N\equiv\times_{n\in\bar{N}}\bar{K}$. The set of pure strategy profiles of all firms other than n is $\bar{K}_{-n}\equiv\times_{n'\in\bar{N}\setminus\{n\}}\bar{K}$. The vector $\hat{N}_{-n}=\left(\hat{N}^1,\hat{N}^2,...,\hat{N}^k,...,\hat{N}^K\right)$ is the distribution of number of firms in each location before firm n decides where $\sum_{k=1}^{K}\hat{N}^k=N-1$, $\hat{N}^k=\sum_{n'\in\bar{N}\setminus\{n\}}1_{l_{n'}=k}$, $\forall k\in\bar{K}$, where $1_{l_{n'}=k}$ is an indicator function taking value 1 if the pure strategy of firm *n* is *k* and zero otherwise. The vector of distribution of firm location after firm *n* selects a location is $\dot{N}=\left(N^{1},N^{2},...,N^{k},...,N^{K}\right)$.

Firm *n*'s mixed strategy is a probability mass function f_{ω_n} of a random variable $\omega_n:\bar{K}_{-n}\rightarrow\bar{K}$ that takes its values from the pure strategy set $\bar{K}$ defined by $f_{\omega_n}\left(l_m\right)\equiv\mu\left(\omega_m=\{l_m\}\right),\ l_m\in\bar{K}$ and $\sum_{l_m\in\bar{K}}f_{\omega_n}\left(l_m\right)=1$.[7] When the context is clear f_{ω_n} is abbreviated as f_n which can also be expressed in vector notation as $\bar{f}_n=\left(f_n(1),f_n(2),...,f_n(k),...,f_n(K)\right)$. The pure strategy $l_n=k$ of player *n* is the mixed strategy f_n such that $f_n(k)=1$ and $f_n(l_n)=0,\ l_n\neq k.$ The set of all mixed strategies of firm *n* is a *K* dimensional simplex[8]

$$\Omega_n=\Delta\left(\bar{K}\right)=\left\{\bar{f}_n=\left(f_n(0),f_n(1),f_n(2),...,f_n(k),...,f_n(K)\right)\in\mathbb{R}^{K}_{\geq 0}:\sum_{k=0}^{K}f_n(k)=1\right\}.$$

The set of all mixed strategy profiles is the Cartesian product of every players' mixed strategy set, $\times_{n\in\bar{N}}\Omega_n=\times_{n\in\bar{N}}\Delta\left(\bar{K}\right)=\Delta\left(\bar{K}_N\right)\equiv\Omega.$ An element of is the set of Ω probability mass functions, or let $f\in\Omega$, then $\bar{f}=\left(f_1,f_2,...,f_n,...,f_N\right),\ f_n\in\Delta\left(\bar{K}\right),\ \forall n\in\bar{N}.$ The set of mixed strategy profiles of all the players other than *n* is denoted by $\Omega_{-n}=\times_{n'\in\bar{N}\setminus\{n\}}\Omega_{n'}$. Let $f_{-n}\in\Sigma_{-n}$, then $\bar{f}_{-n}=\left(f_1,f_2,...,f_{n-1},f_{n+1},...,f_N\right),\ f_{n'}\in\Delta\left(\bar{K}\right),\forall n'\in N\setminus\{n\}.$

5.2 Payoff

Firm *n*'s expected profit when it plays the pure strategy $l_n=k\in\bar{K}$ while other firms play the mixed strategy profile f_{-n} equal to expected profit in the price game, given $N-1$ firms playing the mixed strategy profile $\bar{F}^{*}\left(p:\bar{q}^{*}\right)$ and $E\left(M^{k}\right)$ consumers playing the search game with the mixed strategy profile $\bar{q}^{*}$, or

$$\pi_n\left(k:\bar{f}_{-n},\bar{q}^{*},\bar{F}^{*}\left(p:\bar{q}^{*}\right)\right)=\left(v-r\right)q^{*}\frac{E\left(M^{k}\right)+I_C^{k}}{E\left(\hat{N}^{k}\right)+I_F^{k}+1} \tag{11}$$

Note that $I_F^k,I_C^k=0,\ \forall k\in\bar{K}\setminus\{0\}$. Firm *n*'s BR correspondence is a point-to-set function that maps a mixed strategy profile chosen by all the players other than himself to set of his mixed strategies for the highest payoff, as defined by

$$BR_n\left(\bar{f}_{-n}\right)=\left\{f_n^{*}\in\Omega_n:\pi_n\left(f_n^{*}:\bar{f}_{-n}\right)=\sup_{F_n}\left(f_n:\bar{f}_{-n}\right)\right\},\ \forall\bar{f}_{-n}\in\Omega_{-n}, \tag{12}$$

$BR\left(\bar{f}\right)=\times_{n\in\bar{N}}BR_n\left(\bar{f}_{-n}\right)$ is the set of all mixed strategies BR of the location game.

5.3 Location Equilibrium

When there are pre-existing firms and consumers, $I_F^0,I_C^0=0$, location 0 is just an ordinary location like any other. Therefore, all locations are also identical in the firm perspective. Thus, they play the mixed strategy that gives equal probability to be found at each location *k*, such as $\bar{f}'_n=(f'_n)_{k\in\bar{K}},\ f'_n(k)=\frac{1}{K+1},\ \forall k\in\bar{K},\ \forall n\in\bar{N}.$ Thus the expected number of firms at each location *k* before firm *n* makes the location decision is $E\left(\hat{N}^{k}\right)=\sum_{n'\in\bar{N}\setminus\{n\}}f'_{n'}(k)=\frac{N-1}{K+1},\ \forall k\in\bar{K}.$

---

[7] Let $\left(\bar{K}_{-n},\tilde{K}_{-n},\mu\right)$ be a probability space, $\tilde{K}_{-n}$ is a Borel set satisfying the usual properties, and $\mu:\tilde{K}_{-n}\rightarrow[0,1]$ is a probability measure such that $\mu\left(\bar{K}_{-n}\right)=1$.

[8] The number of pure strategy is $K+1$, so the dimension of the simplex is K.

Let f^* be a mixed strategy profile such that $f^*(0)=x$ and $f^*(k)=(1-x)/K, \forall k \in \bar{K}\setminus\{0\}$ where $Kf^*(k)+f^*(0)=1$. That is a firm playing f^* has a probability *x* that it will be found at location 0 and put an equal probability to locate at the rest. Denote the mixed strategy profile when all firms other than *n* play the same mixed strategy $\bar{f}^*_{-n}$ by $\bar{f}^*_{-n}=(f^*)_{n'\in\bar{N}\setminus\{n\}}$. When all firms other than *n* play the mixed strategy profile $\bar{f}^*_{-n}$ the expected number of firms in location 0 and each of the other locations is

$$E(\hat{N}^0)=\sum_{n'\in\bar{N}\setminus\{n\}} f_{n'}(0)=(N-1)f^*(0)=(N-1)x \tag{13}$$

$$E(\hat{N}^k)=\sum_{n'\in\bar{N}\setminus\{n\}} f_{n'}(k)=(N-1)f^*(k)=(N-1)\frac{1-x}{K+1}, \tag{14}$$

respectively. Firm *n*'s expected payoff when the pure strategy $l_n=0$ and $l_n=k$ is played, given $\bar{f}^*_{-n}$ is

$$\pi_n(0:\bar{f}^*_{-n},\bar{q}^*)=(v-r)q^*\frac{E(M^0)+I^0_C}{E(\hat{N}^0)+I_F+1}=(v-r)q^*\frac{\frac{M}{K+1}+I^0_C}{(N-1)x+I^0_F+1} \tag{15}$$

$$\pi_n(k:\bar{f}^*_{-n},\bar{q}^*)=(v-r)q^*\frac{E(M^k)}{E(\hat{N}^k)+1}=(v-r)q^*\frac{\frac{M}{K+1}}{(N-1)\frac{1-x}{K}+1} \tag{16}$$

The key variable is the ratio of the total number of consumers in the pre-existing market and other locations $k\in\bar{K}\setminus\{0\}$,

$$\Lambda\equiv\frac{E(M^0)+I^0_C}{E(M^k)}=1+(K+1)\frac{I^0_C}{M}. \tag{17}$$

Equating (15) and (16) gives the equal expected profit condition of the location game

$$\pi_n(0:\bar{f}^*_{-n},\bar{q}^*)=(v-r)q^*\frac{E(M^0)+I^0_C}{E(\hat{N}^0)+I^0_F+1}\equiv(v-r)q^*\frac{E(M^k)}{E(\hat{N}^k)+1}=\pi_n(k:\bar{f}^*_{-n},\bar{q}^*) \tag{18}$$

Substituting $E(\hat{N}^0)$ and $E(\hat{N}^k)$ to solve for *x* yields

$$x^*=\frac{\Lambda[(N-1)+K]-K(1+I^0_F)}{(N-1)(K+\Lambda)}=\frac{[(N-1)+K](K+1)I^0_C+M(N-1)-KMI^0_F}{(N-1)(K+1)(M+I^0_C)} \tag{19}$$

Thus, when all firms other than *n* play the mixed strategy profile $\bar{f}^*_{-n}$ with $x=x^*$, firm *n*'s BR is Ω_n since playing any pure strategies gives the same expected utility, or indifference between being founded at location 0 or location $k\in\bar{K}\setminus\{0\}$, equal to

$$\pi_n(\bullet:\bar{f}^*_{-n},\bar{q}^*,\bar{F}^*(p:\bar{q}^*))=(v-r)q^*\frac{M+(K+1)I^0_C}{N+K+I^0_F}\equiv\hat{\pi}. \tag{20}$$

That is, $f^*\in BR_n(\bar{f}^*_{-n})$ and $\bar{f}^*$ is a fixed point of $BR(\bar{f}^*)$, or $\bar{f}^*\in BR(\bar{f}^*)\equiv\times_{n\in\bar{N}}BR_n(\bar{f}^*_{-n})$. Hence, the mixed strategy profile $\bar{f}^*$ of all *N* firms is the MSNE of the location game. When

$I_F^0, I_C^0 = 0$, $\bar{f}^*$ collapses to the symmetric mixed strategy profile, or $\bar{f}^* = \bar{f}' \equiv \left(\bar{f}_n'\right)_{n\in\bar{N}}$ with $x^* = \frac{1}{K+1}$ and $\hat{\pi} = (v-r)q^* \frac{M}{N+K}$.

The comparative statics are reported in Table 1, where the left columns are the interested variables taking the partial derivative with respect to the parameters in the rows. The probability that a firm will locate in the pre-existing market, x^*, which represents the degree of agglomeration, increases with the expected demand in that location and the number of total locations, but decreases with the number of total consumers and firms in that location. Since the increase in the total number of consumers *M* increases the expected number of consumers in each location, the ratio of total number of consumers in the pre-existing market and other locations, Λ, decreases as *M* increases, as does x^*.

The main finding in this paper is that imperfect information decreases the expected demand in each location when the number of total potential locations *K* increases, and stimulates firms to agglomerate in the location where the demand is certain. Since probability cannot exceed one, the threshold is when $K \geq \frac{M}{I_C^0}\left[(N-1)+I_F^0\right]-1$, x^* reaches its upper bound, which is the full agglomeration case where all firms are found in the same location, the pre-existing market.

Table 1 Comparative Statics

	$E(M^0)$	$E(M^k)$	Λ	K	M	N	I_C^0	I_F^0
$E(M^0)$	1	n/a	n/a	-	+	0	+	0
$E(M^k)$	n/a	1	n/a	-	+	0	0	0
Λ	n/a	n/a	1	+	-	0	+	0
x^*	+	-	+	+	-	+/-	+	-

Source: Author

6. Multiple Pre-Existing Markets

In this appendix, we study the case of multiple pre-existing markets. Suppose that there are two pre-existing markets. The location set is now expanded to $\bar{K}' = \{A, B, 1, 2, ..., k, ..., K\}$, $K \in \mathbb{N}$, with a total of $K+2$ locations. Note that location 0 is replaced with location *A* and location *B* According to the location game but using two pre-existing markets, let x_A denote the probability that a firm will locate in pre-existing market *A* and x_B denote the probability that a firm will locate in pre-existing market *B* while $\frac{1-x_A-x_B}{K}$ is the probability that they will locate in each of the other *K* locations. The expected number of firms not including firm *n* in the pre-existing markets *A* and *B* and other location *k* are expressed by

$$E\left(\hat{N}^A : x_A\right) = (N-1)x_A \qquad (21)$$

$$E\left(\hat{N}^B : x_B\right) = (N-1)x_B \qquad (22)$$

$$E\left(\hat{N}^k : x_A, x_B\right) = (N-1)\frac{1 - x_A - x_B}{K}, \tag{23}$$

respectively. The total number of firms in the pre-existing market *A* and *B* equal the expected number of firms who choosing to be found in that locations plus the number of pre-existing firms there, denoted by I_F^A and I_F^B respectively.

The distribution of consumers across all locations is identical because consumers have random uniformity with an equal probability $\frac{1}{K+2}$ to visit any location, since they do not have any prior knowledge of the distribution of firms. Therefore, $E\left(M^k\right) = \frac{M}{K+2},\ k \in \bar{K}'$. The total number of consumers in pre-existing markets *A* and *B* equals to the expected number of consumers who randomly visit the locations plus the number of pre-existing consumers in each location, denoted by I_C^A and I_C^B. The ratio between the total number of consumers in pre-existing market *A* and *B* and other location is $\Lambda_i \equiv \frac{E\left(M^i\right) + I_C^i}{E\left(M^k\right)} = 1 + (K+2)\frac{I_C^i}{M}$, which equals one when $I_C^i = 0,\ i = A, B$.

As in equation (15) and (16)of chapter one, firm *n*'s location decision given the location decisions of all firms other than *n*, can be found by the equal expected profit condition for pre-existing markets *A* and *B* and the other location *k*

$$\pi_n\left(A : \bar{f}_{-n}^*, \bar{q}^*, \bar{F}^*\left(p : \bar{q}^*\right)\right) \equiv \pi_n\left(k : \bar{f}_{-n}^*, \bar{q}^*, \bar{F}^*\left(p : \bar{q}^*\right)\right)$$
$$\frac{E\left(M^A\right) + I_C^A}{E\left(M^k\right)} = \frac{E\left(\hat{N}^A : x_A\right) + I_F^A + 1}{E\left(\hat{N}^k : x_A, x_B\right) + 1} \tag{24}$$

$$\pi_n\left(B : \bar{f}_{-n}^*, \bar{q}^*, \bar{F}^*\left(p : \bar{q}^*\right)\right) \equiv \pi_n\left(k : \bar{f}_{-n}^*, \bar{q}^*, \bar{F}^*\left(p : \bar{q}^*\right)\right)$$
$$\frac{E\left(M^B\right) + I_C^B}{E\left(M^k\right)} = \frac{E\left(\hat{N}^B : x_B\right) + I_F^B + 1}{E\left(\hat{N}^k : x_A, x_B\right) + 1} \tag{25}$$

Next, substitute $E\left(\hat{N}^A : x_A\right)$, $E\left(\hat{N}^B : x_B\right)$ and $E\left(\hat{N}^k : x_A, x_B\right)$ into(24) and (25) to solve for the MSNE x_A^* and x_B^* of the firms

$$x_A^*\left(K, N, M, I_C^A, I_C^B, I_F^A, I_F^B\right) = \frac{\Lambda_A\left[(N-1) + K\right] + \left[\left(\Lambda_A - \Lambda_B\right) - K\right]\left(1 + I_F^A\right)}{(N-1)\left(K + \Lambda_A + \Lambda_B\right)} \tag{26}$$

$$x_B^*\left(K, N, M, I_C^A, I_C^B, I_F^A, I_F^B\right) = \frac{\Lambda_B\left[(N-1) + K\right] + \left[\left(\Lambda_B - \Lambda_A\right) - K\right]\left(1 + I_F^B\right)}{(N-1)\left(K + \Lambda_A + \Lambda_B\right)} \tag{27}$$

When $I_C^A = I_C^B$, $x_A^* = x_B^*$ and both pre-existing markets are identical. When $I_C^A > I_C^B$, the pre-existing market *A* is bigger than *B* and they are heterogeneous in the agglomerated location where *A* has higher expected number of firms than *B*, although all the remaining *K* locations have the same expected number of firms, $\frac{1 - x_A^* - x_B^*}{K}N$.

When $I_C^A = I_C^B = 0$, $x_A^* = x_B^* = \frac{1}{K+2}$ and there is no agglomeration since all locations are identical. In fact, since firm location distribution is a function of consumer location distribution, the analysis could be extended to the case of *K* pre-existing markets. That is, pre-existing firms and consumers $I_F^k, I_C^k > 0,\ \forall k \in \bar{K}$ could be introduced. Then, finding the MSNE of the location game involved solving the system of *K* equations.

7. Conclusions and Future Research

This paper presents a new way to explain the location decisions of firms regardless of the distance between them when no locations are closer to consumers than any others. In conventional models, distance between firms and consumers determines the expected demand of each firm. Here, the number of the possible locations determines the expected demand of each location. The increase in number of locations increases the number of available options for consumers to choose at random and lowers the expected demand for each location. As a result, demand uncertainty caused by consumer imperfect information is the source of firm agglomeration in this model.

This paper also presents a tradeoff for firms in choosing between a location with high competition but certain demand guaranteed or locations with low competition but uncertain demand. When applying this model to the real world, a firm facing a location problem has to consider the total number of potential locations that consumers are aware of. If there are a large number of potential locations, the chance that a particular location will be visited is considerably lower. Hence, it would be reasonable to choose a location with high competition, but with a sufficient number of active customers of firms already established. This tradeoff explains the phenomenon for a market of various kind of products that usually have a number of firms agglomerated in the same location or market, such as computers, automobiles, jewelry, souvenirs, and musical instruments.

Since the model in this paper is fairly simple, it has a few drawbacks. First, since all locations are identical, firms have equal probability to be found at each location. Therefore, their expected number in each location is identical. This is in contrast to the real world, where the number of firms in each location is rarely equal. Second, the number of firms has no effect on the price offered in each location, so the expected prices in the pre-existing market and other markets is the same. Therefore, the expected utility of consumers is the same, whether or not they visited any locations.

To address these issues, the followings suggestions are made for future research: first, the case of multiple pre-existing markets is introduced to create more heterogeneity in the number of firms at each location, which brings the model closer to the real world. Second, the heterogeneous search cost is introduced to have aheterogeneous expected price at each location. This subject is dealt with my dissertation. Pre-existing consumers are reintroduced as local consumers with lower search costs, while ordinary consumers are assumed to be foreigners with higher search costs. Since a higher number of low cost consumers induces a higher degree of price comparison and increases the weight that the firms put on them, the higher the ratio between the number of local and foreign consumers, the lower the expected price

Search cost heterogeneity can generate a relationship between equilibrium price distribution and the number of firms, if the number of firms can affect the consumer search cost. In subsequent work by the author, density dependent search costs are introduced where the location with a denser crowd of firms results in a lower price search cost. This assumption creates a difference in expected price in each location. This leads consumers to conduct location searches for the lowest price location. Search methods for location searches are also introduced, called two-step searching, which doubles the process of the non-sequential search. As a first step, consumers search for a location with the highest utility by anticipating the outcome of the price search in the second step. With location search, consumer location distribution is now endogenized. This presents a feedback effect and enhances the agglomeration force. Therefore, this paper may be considered as a stepping stone for future research.

Acknowledgement

This paper is based on part of my Ph.D. dissertation at the Faculty of Economics, Thammasat University. I am indebted to my dissertation committee for guidance and support. I thank two anonymous referees for valuable comments. This paper benefited from the Ph.D. Student Academic Co-Workshop at Chulalongkorn University, December 2013, and the Eighth Conference on Economics for Graduate Students, Chiang Mai University, July 2014. The author gratefully acknowledges the financial support provided by Thammasat University Research Fund under the TU Research Scholar, Contract No. ทบ. 2/2557

References

Baye, M. R., Morgan, J., & Scholten, P. (2006).Information, search, and price dispersion. Handbook on Economics and Information Systems, 1.

Burdett, K., & Judd, K. L. (1983).Equilibrium price dispersion. Econometrica, 51(4), 955-969.

Burdett, K., & Smith, E. (2010).Price distributions and competition. Economics Letters, 106(3), 180-183.

Diamond, P. A. (1971). A model of price adjustment. Journal of Economic Theory, 3(2), 156-168.

Hotelling, H. (1929). Stability in Competition. Economic Journal, 39(153), 41-57.

Janssen, M. C., Moraga-González, J. L., & Wildenbeest, M. R. (2005). Truly costly sequential search and oligopolistic pricing. International Journal of Industrial Organization, 23(5), 451-466.

Janssen, M. C., & Moraga-González, J. L. (2004). Strategic pricing, consumer search and the number of firms. The Review of Economic Studies, 71(4), 1089-1118.

Krugman, P. (1991). Increasing Returns and Economic Geography. The Journal of Political Economy, 99(3),483-499.

Moraga-Gonzalez, J. L., Sandor, Z., & Wildenbeest, M. R. (2010).Non-sequential searchequilibrium with search cost heterogeneity (No. D/869). IESE Business School. Stahl, D. O. (1989).Oligopolistic pricing with sequential consumer search. The American Economic Review,79(4), 700-712.

Rothschild, Michael and Stiglitz, Joseph E., (1970), Increasing risk: I. A definition, Journal of Economic Theory,2, issue 3, p. 225-243.

Stigler, G. J. (1961). The economics of information. The Journal of Political Economy, 69(3), 213-225.

Takahashi, T. (2013). Agglomeration in a city with choosy consumers under imperfect information. Journal of Urban Economics, 76, 28-42.

Weitzman, M. L. (1979). Optimal search for the best alternative. Econometrica, 47(3), 641-654.