



# On The Effect of Money on Growth within an Optimal Control Framework

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## Abstract

This study addresses a gap in the monetary policy literature by examining the dynamic relationships between money supply, the money-interest rate sensitivity parameter, and key economic indicators, including consumption, money demand, capital, price, and interest rates, using an optimal control framework and Hamiltonian approach. Crucially, the model incorporates money as a factor in the Cobb-Douglas production function, alongside capital and labor, and as a component of the utility function, to offer a holistic view of money's role in the economy. The key findings derived from the simulation results show a positive relationship between money supply and capital, confirming that monetary expansion can stimulate economic growth. It also reveals an inverse relationship between the money supply and price after the initial interval, which contradicts the traditional inflation expectations of the Quantity Theory of Money. Furthermore, the study identifies that when the money-interest rate sensitivity parameter is set high, it exhibits volatile dynamics across all the variables. Therefore, while expansionary monetary measures can stimulate economic growth, policymakers must design optimal levels of money supply and money-interest rate sensitivity parameters to achieve economic stability and growth.

**Keywords:** Money Supply, Capital Accumulation, Interest Rate, Price, Optimal Control, Hamiltonian

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## **1. Introduction**

Money supply, a fundamental concept in economics, refers to the amount of money available in an economy at a given time. It plays a pivotal role in shaping economic outcomes, particularly in influencing inflation and economic growth. The relationship between money supply and the economy is multifaceted and complex, with its effects often manifesting after a time lag (Kemal, 2006). Money supply serves as a crucial tool for central banks in their pursuit of economic objectives, such as price stability and sustainable growth. By manipulating the money supply, central banks can influence interest rates, which, in turn, affect borrowing, spending, and investment patterns across the economy. However, the effectiveness of these policies can vary depending on the economic context and specific implementation approach (Keister et al., 2008).

Understanding the impact of money supply provides several benefits for economic policymaking and analysis. It allows policymakers to better predict and manage inflation, which is crucial for maintaining price stability and preserving the purchasing power of the currency (Kemal, 2006; Qayyum, 2022). In addition, knowing this impact helps manage interest rates and, consequently, economic growth (Christiano & Eichenbaum, 1992). Thus, studying the impact of money supply has attracted numerous scholars and invited them to employ various methodologies to unravel such impacts. For example, Gavin & Kydland (1995) employed a business cycle model to study how alternative money supply rules affect observed business cycle facts in the United States. Their model demonstrates that changes in the money supply significantly impact the cyclical nature of nominal variables but have a minimal effect on real variables. Liu et al. (2022) used panel data and econometric techniques, including the generalized method of moments (GMM), to investigate the relationship between money supply and bank stock returns in South Asian countries. Their findings reveal a positive association between the money supply and bank stock prices. In the context of developing countries, Chindengwike (2022) employed time-series data and the Auto Regressive Distributed Lag Model (ARDL) to demonstrate that money supply has a significant positive effect on economic growth. Brzezinski et al. (2024) took a particularly innovative approach in their study of the Spanish Empire from 1531 to 1810. Using a natural experiment approach, this study found that negative money supply shocks caused a decline in real output, highlighting slow price adjustments and credit frictions as transmission mechanisms.

While previous studies have provided valuable intuitions, there remains a need for a better understanding of how the money supply interacts with various economic variables in a dynamic setting. Specifically, the relationships between money supply, interest rates, prices, and capital have not been explored in a unified framework. One key gap is the lack of a model that incorporates money as both a factor in the production function and a component of the utility function within an optimal control framework. While some studies have explored money as a factor of production (e.g., Sinai & Stokes (1972), Benchimol (2014) and others have examined money demand in utility functions (e.g., Sidrauski (1967); Poterba & Rotemberg (1986); Ivanov et al. (2014)), few have combined these approaches to provide a holistic view of the role of money in the economy. In addition, there is a need to reconcile the sometimes-contradictory findings in the literature regarding the role of money in economic models. For instance, while some studies find a significant role for money in production functions, others, such as Benchimol & Fourçans (2011), suggested a minor role in certain contexts. Thus,

understanding these discrepancies and their implications for monetary policy is crucial. This study contributes to the existing literature by addressing these gaps through a potential methodological approach and an analysis of key economic variables. The remainder of this work is organized as follows to achieve our objectives. Section 2 surveys the relevant literature, examining the various impacts of the money supply on key economic variables. In Section 3, we present the mathematical model that forms the basis of our analysis. Subsequently, we present the results, followed by a discussion and policy implications.

## **2. Literature Reviews**

This section explores four major theoretical frameworks that have shaped our understanding of the relationship between money supply and various economic indicators, including the Quantity Theory of Money (QTM), Monetarist Theory, New Keynesian Theory, and Austrian Business Cycle Theory (ABCT). This review also examines empirical research to understand the impact of the money supply on the economy.

### **2.1 Quantity Theory of Money**

The Quantity Theory of Money (QTM) is a cornerstone concept in monetary economics that establishes a fundamental relationship between the money supply and price levels in an economy. This classical theory, with roots tracing back to the 16th century and formalized by Irving Fisher in the early 20th century, posits that an increase in the money supply leads to a proportional increase in the general price level in the long run (Kemal, 2006; Teles et al., 2016). Fundamentally, the QTM is expressed through the equation of exchange,  $MV = PT$ , where  $M$  represents the money supply,  $V$  is the velocity of money,  $P$  denotes the price level, and  $T$  stands for the volume of transactions or output. This equation suggests that changes in the money supply directly influence price levels, assuming that the velocity and output remain constant. The theory implies that inflation is primarily a monetary phenomenon, with changes in the money supply being the main driver of price changes (Kemal, 2006).

However, a careful reading of the literature shows that this “organizing principle” is neither universally applicable nor empirically inviolable, and modern scholarship increasingly treats it as a useful benchmark whose predictive power depends on institutional context, policy regime, and the proper treatment of velocity and output dynamics. Early critiques have highlighted its shortcomings. Smith (1985) treats colonial America as a natural experiment in which large swings in the supply of paper money frequently failed to translate into proportional movements in prices or in exchange rates. In several colonies, wartime note issues soared to unprecedented per-capita levels, but price levels rose only modestly or even fell, and the dispersion in colonial exchange rates bore little systematic relation to local money stocks. Smith attributes this apparent “failure” to the fact that colonial notes were often backed by future tax revenues, making their value contingent on fiscal surpluses rather than on their sheer quantity. Accordingly, when money injections were credibly matched by future revenue streams, expectations of redemption stabilized prices despite rapid monetary expansion, implying that an “unbacked” version of the QTM is merely a special case rather than the general rule (Smith, 1985).

Contrasting evidence is provided by modern cross-country studies that focus on longer horizons and more inflation-prone environments. Alvarez et al. (2001) reaffirmed the QTM as a “long-run standard,” arguing that average money growth tracks average

inflation and nominal interest rates remarkably well once one looks across decades and national experiences characterized by substantial variation in monetary policy. For instance, the surge in U.S. money growth during the 1970s and the early 1980s coincided with persistently high inflation and interest rates, whereas the subsequent slowdown in money growth accompanied the disinflation of the 1990s. Their segmented-markets model formalizes this intuition: equilibrium prices satisfy a variant of the equation of exchange, while the Fisher relation ensures that nominal interest rates equal the real rate plus the ongoing rate of money creation. Short-run liquidity effects allow central banks to manipulate interest rates without immediately violating the quantity-theoretic discipline; however, over longer windows, controlling the money stock remains the only reliable lever for anchoring inflation expectations (Alvarez et al., 2001).

Evidence from developing or inflationary environments continues to support the monetarist proposition that sustained monetary expansion is ultimately inflationary in nature. Using long-time series data and cointegration techniques, Kemal (2006) finds that in Pakistan, the coefficient linking money growth to prices is not statistically different from unity, confirming the long-run validity of the QTM. Although the study detects a lag of roughly three quarters before money growth is fully reflected in prices, the long-run proportionality remains intact, underscoring that inflation is fundamentally a monetary phenomenon over sufficiently extended horizons (Kemal, 2006).

More recent empirical work probes why the raw correlation between money growth and inflation seems to weaken in low-inflation, inflation-targeting regimes. Teles et al. (2016) started from the observation that, for countries operating near price stability, scatter plots of average money growth versus inflation often look like random noise. However, they show that this breakdown is largely an artifact of ignoring two theoretically motivated adjustments. First, real output growth must be netted out because rising real GDP can absorb growing nominal balances without increasing prices. Second, velocity is endogenous to the opportunity cost of holding money; when nominal yields fall, desired money balances rise, thereby weakening the raw money-inflation link. After correcting for these effects using elasticities derived from Baumol–Tobin and Miller–Orr money-demand models, the seemingly erratic relationship “lines up nicely” along a 45-degree line, restoring the one-to-one prediction even for the 1970–2005 sample that includes many low-inflation episodes. The apparent deterioration of the QTM under explicit inflation targeting thus reflects policy success—central banks engineered such small inflation differentials that econometric identification became difficult—rather than theoretical invalidation (Teles et al., 2016).

These studies paint a portrait of QTM. On one hand, historical episodes such as colonial America show that institutional details—particularly credible fiscal backing—can insulate prices from money supply shocks, violating the simplistic “money causes inflation” mantra. However, modern cross-country evidence and econometric analyses reaffirm that once velocity determinants and output growth are properly accounted for, persistent deviations between money growth and inflation are rare. In other words, the QTM continues to be a powerful long-run equilibrium condition, but its empirical performance hinges on recognizing that velocity is neither constant nor exogenous and that fiscal-monetary interactions can offset or amplify monetary impulses. Consequently, policymaking frameworks that rely solely on short-term interest rate targeting must still take aggregate money dynamics seriously if they wish to secure low and stable inflation over the long haul.

## **2.2 Monetarist Theory**

Monetarist theory, a significant school of economic thought primarily associated with Nobel laureate Milton Friedman and the Chicago School of Economics, emerged as

a powerful counterpoint to Keynesian economics in the mid-20th century. Fundamentally, monetarism emphasizes the critical role of the money supply in shaping economic outcomes, particularly inflation and economic growth (Andersen & Carlson, 1970). The central tenet of this theory is that changes in the money supply are the primary determinants of economic activity and price levels, asserting a direct and predictable relationship between money supply growth and inflation, especially in the long run (Qayyum, 2022). Monetarists argue that inflation is fundamentally a monetary phenomenon resulting from an increase in the money supply that outpaces economic growth. This perspective led to a focus on monetary policy as the primary tool for economic stabilization, in contrast to the Keynesian emphasis on fiscal measures (Ali & Irum, 2022; De Long, 2000). The theory is grounded in the Quantity Theory of Money, expressed through the equation  $MV = PY$ , which establishes a direct link between the quantity of money and price levels. Monetarist theory fleshes out  $MV = PY$  by adding assumptions about how people behave, how policies work, and what happens in the real world, transforming a simple accounting identity into a predictive model of inflation and a normative blueprint for monetary policy. Monetarists also introduced concepts such as the long-run neutrality of money and the natural rate of unemployment, suggesting that attempts to push unemployment below its natural rate through monetary policy would only lead to higher inflation. Although pure monetarism has fallen out of favor in recent decades, its influence remains pervasive in modern macroeconomics. Many of its key insights have been incorporated into mainstream economic thinking, even if they are not explicitly labeled as "monetarist" (De Long, 2000). The theory's impact on economic policy was substantial, particularly in the 1980s, when it influenced central banks to prioritize inflation control through the management of the money supply. However, the ongoing debate between monetarist and Keynesian approaches highlights the complexity of economic management, with some economists advocating a balanced approach that considers both monetary and fiscal policies (Ali & Irum, 2022). Despite facing criticisms regarding the stability of the money supply, economic variable relationships, and challenges in defining and controlling money supply in modern financial systems, monetarist theory remains crucial for understanding the role of money in the economy and conducting monetary policy.

### **2.3 New Keynesian Theory**

New Keynesian Theory, a prominent school of macroeconomic thought that emerged in the 1980s, represents a significant evolution in economic thinking by bridging traditional Keynesian ideas with modern macroeconomic modeling techniques. This theory aims to provide a more comprehensive understanding of economic fluctuations and policy implications by incorporating elements of both Keynesian and classical economics (Satti et al., 2007). According to Satti et al. (2007), the New Keynesian Phillips Curve (NKPC) is a fundamental component of the broader New Keynesian economic framework. This approach represents a new neoclassical synthesis that integrates traditional Keynesian features, such as nominal rigidities and imperfect competition, with a dynamic stochastic general equilibrium (DSGE) model built upon micro-foundations and rational expectations. The NKPC is an inflation adjustment equation derived from the profit-maximizing behavior of firms within a monopolistically competitive market. A key distinction from the traditional Phillips curve is its forward-looking nature, where current inflation is dependent on the stream of future expected real marginal costs. This allows for the analysis of deep structural parameters such as the degree of price inertia and discount factor. A significant implication for monetary policy is that a credible announcement from a central bank can shape the future expectations of private agents, which, in turn, affects current inflation. This suggests that, unlike

traditional models, disinflationary policy does not need to be costly. The New Keynesian Theory seeks to explain why markets might not clear instantly and why monetary policy could have real effects on the economy in the short run, even when economic agents are acting rationally. Key features of New Keynesian Theory include nominal rigidities, including price and wage stickiness; imperfect competition; rational expectations; coordination failures; efficiency wage theory; and real effects of monetary policy. While the New Keynesian Theory has become dominant in macroeconomics and significantly influenced policymaking, it has not been without criticism. Some economists argue that it has adopted many views once considered monetarist or classical, leading to questions about the appropriateness of the term "Keynesian" (Mankiw, 1992). Furthermore, the theory has faced challenges in fully explaining phenomena such as the 2007-2008 financial crisis and subsequent economic stagnation (Galí, 2018; Palley, 2023). Despite these criticisms, the New Keynesian Theory remains influential in academic research and policy modeling, providing valuable insights into the workings of the economy. However, ongoing debates and criticisms highlight the need for the continued refinement and development of macroeconomic theory to address complex economic realities and emerging challenges (Galí, 2018; Palley, 2023).

#### **2.4 Austrian Business Cycle Theory**

The Austrian Business Cycle Theory (ABCT), developed by prominent economists of the Austrian School, such as Ludwig von Mises and Friedrich Hayek, offers a distinctive explanation for economic fluctuations, focusing on the role of monetary interventions in creating business cycles. Fundamentally, ABCT posits that the manipulation of interest rates and money supply by central banks leads to a misallocation of resources, resulting in unsustainable economic booms followed by inevitable busts (Espinosa et al., 2023). The theory begins with the concept of a "natural" interest rate, which would emerge in a free market based on the time preferences of economic actors. When central banks artificially lower interest rates below this natural level, typically through expansionary monetary policy, it sends false signals to the market, encouraging businesses to invest in long-term capital-intensive projects that appear profitable under these artificial conditions (Howden, 2009). This leads to what Austrian economists' term "malinvestment," particularly in interest-rate-sensitive industries. However, this boom is unsustainable as it is not backed by a genuine increase in savings or real resources in the economy. As businesses compete for scarce resources, prices rise, especially for higher-order goods, eventually revealing the unprofitability of many investments given real consumer preferences and resource availability. The boom then turns to bust as these malinvestments are liquidated, resulting in a recession that Austrian economists view as a necessary, although painful, correction process (Huerta de Soto et al., 2021). Empirical support for this theory comes from an analysis of the True Money Supply (TMS) performance in the United States between 1975 and 2022, which demonstrates the central banking system's role in causing business cycles (Espinosa et al., 2023). However, contrary to some interpretations, the Hayek-Robbins version of ABCT did not advocate a policy of liquidationism during the Great Depression (White, 2008). ABCT's implications include a critique of expansionary monetary policies, a focus on capital structure distortions, an endogenous explanation for business cycles, and policy recommendations favoring minimal government intervention in monetary affairs. While the theory offers valuable insights into the potential unintended consequences of monetary policy, it faces criticism from mainstream economists, who argue that it may oversimplify complex economic phenomena. Nevertheless, ABCT remains a significant perspective for understanding the dynamics of business cycles and the role of monetary policy in economic stability.

The effects of money supply on economic variables have been the subject of extensive research, revealing complex and multifaceted relationships that have evolved over time. Early studies, such as Ulrich and Wachtel (1984), demonstrate that unanticipated increases in the money supply have an immediate positive effect on short-term interest rates, while also identifying a liquidity effect on interest rates occurring weeks before information announcements. Christiano & Eichenbaum (1992) investigate the intricate relationship between the money supply and interest rates, focusing on a phenomenon known as the liquidity effect. The central argument and key empirical finding presented are that an unexpected, or "exogenous," positive shock to the money supply results in a significant and persistent decrease in the short-term interest rates. This conclusion is supported by an analysis of post-war U.S. economic data, which reveals that the choice of monetary aggregates is crucial for observing this effect. The authors find that while broader measures of money supply, such as the monetary base (M0) or M1, often replicate the traditional finding of interest rates rising after a money supply shock, a more direct measure of monetary policy action, that is, innovations to Nonborrowed Reserves (NBR), tells a different story. When using NBR, the data show that exogenous increases in the money supply lead to sharp and lasting declines in the federal funds rate. Quantitatively, the study estimates that a one percent unexpected increase in the money supply can cause the quarterly federal funds rate to fall by a substantial margin, ranging from 26.7 to 53.3 basis points. To explain this empirical result, which contradicts many standard quantitative models that predict an interest rate increase due to anticipated inflation, the authors propose a general equilibrium model with two key features. The first is heterogeneity, which posits that money injections do not affect all economic agents uniformly; instead, firms and financial intermediaries absorb a disproportionately large share of new liquidity. The second feature is ex post inflexibility in production, which means that firms pre-commit to their production plans and cannot easily alter them in the short run. This inflexibility results in a very low short-run interest elasticity of the demand for money, which in turn amplifies the impact of a money supply shock, causing the observed large drop in interest rates. Although mainstream economic theory generally posits an inverse relationship, where an increase in the money supply leads to lower interest rates due to excess liquidity, empirical and contextual findings indicate scenarios where a positive correlation can emerge. For instance, an analysis of monetary policies in Indonesia showed that the BI interest rate has a significant positive correlation with the money supply (M2). The study highlighted that this could be linked to fluctuations in foreign exchange rates, the balance of payments, and government debt factors, which influenced monetary policy effectiveness and led to non-standard relationships between interest rates and money supply (Azmy, 2014). Another important insight comes from the Chinese financial system, where funding liquidity (measured by M2) strongly influences money market interest rates in interactions mediated by market liquidity. This finding suggests that in some financial markets, an increased money supply can be accompanied by higher interest rates due to liquidity demand and supply dynamics affecting funding costs (Dai, 2014). Similarly, research from Malaysia indicates a positive nexus between monetary growth, interest rates, and economic growth, where interest rates and money supply both exhibit positive relationships with economic growth. This implies that in certain developing economies, monetary policy instruments, including interest rates and money supply, may jointly rise in conjunction with efforts to stimulate growth, reflecting a positive relationship between the two variables (Akalpler & Duhok, 2018). In addition, variations occur due to institutional, market, and policy conditions that can cause interest rates to rise alongside the money supply, particularly in economies with volatile external conditions, government debt pressures, or when monetary expansions are associated with inflation

expectations that push interest rates higher (Mahmood et al., 2017; Daoud & Al-Ezzi, 2023).

Jones & Manuelli (1995) established a well-known relationship between money supply and inflation, showing that higher money growth rates lead to higher inflation rates. Geromichalos et al. (2007) further expanded our understanding by linking increases in money supply to higher asset prices, explaining this through the negative relationship between inflation and asset returns. As research progressed, Gavin & Kydland (1995) provided insights into the differential impacts of money supply changes on real and nominal variables, noting that alterations in money supply rules significantly affect the cyclical nature of nominal variables while minimally affecting real variables. Choi & Oh (2000) investigated the effect of the money supply process on the money demand. The author's theoretical framework derives a money demand function that specifically includes a money supply factor in addition to conventional variables. This derivation demonstrates that the money supply process affects the money demand. Specifically, the model suggests that when investors anticipate that the Federal Reserve will tighten its policy, they have a preemptive motive to increase their money demand to mitigate the resulting impact on their money balances. Kemal (2006) and Qayyum (2022) reaffirmed the positive correlation between money growth and inflation, while also noting a lag of several months in this relationship. Buthelezi (2023) introduced the concept of state-dependent effects, showing that the impact of the money supply on GDP and inflation can vary across different economic states and time horizons. The most recent research has further illuminated the effects of the money supply. Madurapperuma (2023) confirmed the positive association between money growth and inflation in Sri Lanka, while Adjemian et al. (2023) observed a stronger correlation between money supply and food price increases than historically noted. Brzezinski et al. (2024) provided historical evidence of how negative money supply shocks can cause real output to decline, using data from Spanish Empire maritime disasters. Sumi & Rabeta (2024) demonstrated the positive relationship between broad money supply and stock market performance in emerging markets like Bangladesh. Oyadeyi et al. (2024) and Oyadeyi (2024) revealed non-linear and asymmetric effects of money supply changes on various economic indicators in Nigeria, emphasizing the need for context-specific monetary policy approaches.

### **3. Methodology**

The concept of money as a factor in the production function has been the subject of debate and research in economics. Several studies have explored this idea with varying conclusions. Some researchers argue that real money balances are a factor of production. Sinai & Stokes (1972) reported significant findings supporting this hypothesis, showing that real money balances enter significantly into a Cobb-Douglas production function fitted to annual data for the United States economy from 1929 to 1967. Similarly, Benchimol (2014) proposed a model in which real money balances enter the production function, although their Bayesian analysis weakens the hypothesis that money is a factor of production. Building on this existing literature, the proposed model considers an economy with a large number of infinitely lived households and firms, with no population growth and no new firms entering or leaving the market. In this model, a representative firm produces a single composite good using the following Cobb-Douglas function:

$$Y = A(K)^{\alpha_1}(N)^{\alpha_2}(M)^{\alpha_3}, \quad (1)$$

where  $Y$  represents the total output.  $A$  refers to total factor productivity, indicating how efficiently inputs are transformed into outputs. The inputs include  $K$ , which represents capital, and  $N$ , which denotes labor or the workforce involved in the production.  $M$  signifies money, highlighting the financial liquidity that supports the production.  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  measure the sensitivity of the output to a change in each input. The sum of these parameters can provide an interpretation of the returns to scale in the production process.

The capital accumulation in this economy can be expressed as follows:

$$\dot{K} = (1 + R)Y - C - I - B, \quad (2)$$

where  $R$  denotes the interest rate.  $C$  represents consumption, which diminishes the available capital because these resources are used in consumption processes. The term  $I$  stands for investment, with the specific relationship  $I = \delta K$ , where  $\delta$  is the depreciation rate. Finally,  $B$  represents the money demand, which accounts for the financial resources held in liquid form.

The evolution of the interest rate  $R$  is expressed as

$$\dot{R} = sP + rR - mM, \quad (3)$$

where  $P$  is the price.  $s$ ,  $r$ , and  $m$  are positive constants. In this model,  $m$  is the money-interest rate sensitivity parameter that captures how strongly the central bank allows the money supply to influence changes in the policy rate.

The concept of  $m$  is adapted from the conceptual framework outlined by Bernanke & Mihov (1996), who identified parameter  $\alpha$  as representing the impact of the money supply on the policy rate. This coefficient is formally defined within the central bank's loss function as the relative weight placed on the expected squared deviations of money growth from its target. Its role becomes explicit in the derived equation for optimal policy innovation, where  $\alpha$  directly governs how the interest rate should respond to revisions in the money growth forecast. The interpretation of these parameters is straightforward. If  $\alpha$  is set to zero, the model describes a pure inflation targeter, meaning that the policy rate does not react to changes in forecasted money growth as long as the inflation forecasts remain constant. However, if  $\alpha$  is greater than zero, it signifies a money targeter, where the interest rate responds to shifts in forecasted money growth, independently of any changes in inflation forecasts. Therefore, the magnitude of  $\alpha$  dictates the sensitivity of the policy rate to money supply movements, with a larger value indicating a more aggressive interest rate response to revisions in the money supply growth forecast. Therefore, adjusting  $\alpha$  is fundamentally an act of changing the central bank's policy objectives or preferences. However, when the parameter  $\alpha$  is too high, it signifies that the central bank places an excessively strong weight on stabilizing money growth, potentially at the expense of its primary objectives, such as price stability or output stabilization. In such a scenario, the central bank's policy instrument, the interest rate, would become highly sensitive to any revisions or innovations in the money supply growth forecast. Because the optimal interest rate adjustment is directly proportional to  $\alpha$ , a very high value would cause substantial and rapid changes in interest rates in response to even minor fluctuations in the money supply, regardless of the stability of the inflation forecast. This effectively transforms the central bank into a rigid money targeter, shifting its focus away from inflationary concerns. This rigidity poses significant risks.

Historically, rigid money targeting has been considered a "second-best" solution, particularly because the relationship between monetary aggregates and prices can become unstable. If the velocity of money is volatile, for instance, then money growth becomes a poor signal for future inflation, and a policy fixated on it (a high  $\alpha$ ) would be inefficient and potentially detrimental to the economy. That is, an excessively high  $\alpha$  could lead to a destabilizing policy rate and harm overall economic performance, especially if the link between money growth and inflation is weak or unstable.

Therefore, in this study,  $m$  is used to measure how aggressively the central bank adjusts interest rates in response to fluctuations in monetary aggregates. Consequently, a high value of  $m$  signifies large policy moves driven by changes in money supply. A central bank can alter this coefficient by strengthening or weakening the mechanical pass-through from money expansion to rate adjustment. This coefficient,  $m$ , ultimately reflects the central bank's prevailing philosophy and operational toolkit.

The evolution of  $P$  is as follows:

$$\dot{P} = aY + bP - dR - hB, \quad (4)$$

where  $a$ ,  $b$ ,  $d$ , and  $h$  are the positive constants.

Regarding money in the utility function, this study posits that within a well-established research tradition. The theoretical foundation was established by Sidrauski (1967), who posited that real money balances directly enhance household welfare and should, therefore, be included as an argument in the utility function. Sidrauski's model is instrumental for deriving and simulating key macroeconomic properties, most notably the superneutrality of money. This property implies that, in the long run, real variables such as capital stock, output, and consumption remain independent of the nominal money growth rate, a result that holds particularly when consumption and money are separable in the utility function. Building on this, Poterba & Rotemberg (1986) provided a notable empirical implementation, which they termed the asset-in-the-utility-function model. They estimated the parameters of an aggregate utility function that incorporates both consumption and liquidity services derived from a portfolio of assets, including real money, time deposits, and short-term government debt, to study household asset demand. Benchimol & Fourçans (2011), using a New Keynesian DSGE framework for the Eurozone, found that money plays a minor role in their estimated model. They note that the impact of money on output depends on the degree of risk aversion. More recently, Ivanov et al. (2014) explicitly applied Sidrauski's framework and calibrated the model to data from Macedonia. Their simulations reaffirmed the model's central property of superneutrality, demonstrating that the real variables in their model were unaffected by monetary shocks. Informed by this knowledge, this work assumes that the central planner maximizes the lifetime utility  $U$  given by:

$$U(C) = \frac{(C(B)^\phi)^{1-\sigma}}{1-\sigma}, \quad (5)$$

where  $\phi$  is the elasticity of utility with respect to money demand, and a value  $>1$  means utility rises more than proportionally with money demand, so households strongly value money, which provides liquidity.  $\sigma$  serves as a measure of the elasticity of intertemporal substitution, indicating how households are willing to adjust consumption across different periods. When  $\sigma = 1$ , preferences are logarithmic, meaning utility is responsive to proportional changes in consumption. If  $\sigma$  is less than 1, it suggests higher elasticity,

implying that the planner is more inclined to substitute consumption over time, showing greater flexibility in adjusting consumption across various time frames. Conversely, a value of  $\sigma$  greater than 1 reflects increased risk aversion and a stronger preference for maintaining stable consumption. This formulation was optimized to achieve maximum lifetime satisfaction.

Using an optimal control framework, the central planner solves the following Hamiltonian problem:

$$H = \frac{(CB^\phi)^{1-\sigma} e^{-\rho t}}{1-\sigma} + \lambda_1 (\dot{K}) + \lambda_2 (\dot{P}) + \lambda_3 (\dot{R}), \quad (6)$$

which provides the following results:

$$\frac{d}{dt} \frac{\lambda_1(t)}{\lambda_1} = - \frac{M^{\alpha_3} N^{-\alpha_2} \left( (1+R)e^{-\rho t} (CB^\phi)^{-\sigma} B^\phi + \frac{e^{-\rho t} (CB^\phi)^{-\sigma} (CB^\phi \phi - B^{1+\phi})a}{hB} \right) A \alpha_1 K^{\alpha_1-1}}{e^{-\rho t} (CB^\phi)^{-\sigma} B^\phi} + \delta, \quad (7)$$

$$\frac{d}{dt} \frac{\lambda_2(t)}{\lambda_2} = \frac{-sN^{-\alpha_2} M^{\alpha_3-1} \alpha_3 A (hR - a + hB)B + Ca\phi) K^{-\alpha_1} - mb(C\phi - B)}{m(C\phi - B)}, \quad (8)$$

$$\frac{d}{dt} \frac{\lambda_3(t)}{\lambda_3} = \frac{mK^{-\alpha_1} N^{-\alpha_2} d(C\phi - B)M^{-\alpha_3+1} - (r(C\phi - B)a + (1+R)hB)\alpha_3 + BMmh)A}{\alpha_3 A((C\phi - B)a + (1+R)hB)}, \quad (9)$$

$$\frac{d}{dt} \frac{C(t)}{C(t)} = - \frac{(-1+\sigma)\phi \left( \frac{d}{dt} \frac{B(t)}{B(t)} \right)}{B(t)\sigma} - \frac{\rho}{\sigma} + \frac{-\alpha_1 M^{\alpha_3} (((1+R)h - a)B + Ca\phi)AN^{-\alpha_2} K^{\alpha_1-1} + \delta hB}{\sigma B h}, \quad (10)$$

$$\frac{d}{dt} \frac{B(t)}{B(t)} = \frac{1}{m(-2 + (\phi - 2)\sigma^2 - 4\sigma)hB(C\phi - B)\phi} \left( \begin{aligned} &3N^{-\alpha_2} m(((1+R)h - a)B + \\ &Ca\phi)M^{\alpha_3} (C\phi - B)A \left( \sigma - \frac{\rho}{2} + \frac{3\delta}{4} - \frac{b}{4} \right) \alpha_1 K^{\alpha_1-1} \\ &- hB \{ N^{-\alpha_2} K^{-\alpha_1} s \alpha_3 (((1+R)h - a)B + Ca\phi)AM \} \end{aligned} \right), \quad (11)$$

$$\begin{aligned} \frac{d}{dt} \frac{M(t)}{M(t)} = & - \frac{\alpha_1 ((1+R)AK^{\alpha_1} N^{\alpha_2} M^{\alpha_3} - C - \delta K - B)}{(\alpha_3 - 1)K} - \frac{\alpha_1 K^{-\alpha_1} N^{-\alpha_2} (\alpha_3 - 1)K((1+R)h - a)B + Ca\phi)}{(\alpha_3 - 1)m(C\phi + B)} \\ & - dmK^{-\alpha_1} M^{-\alpha_3+1} N^{-\alpha_2} (-C\phi + B) \\ & + \frac{-mM(C\phi + B) - mM(-sP - rR)}{(\alpha_3 - 1)R} + \frac{-(r((C\phi - B)a + (1+R)hB)\alpha_3 + BMmh)A}{(\alpha_3 - 1)\alpha_3 A((C\phi - B)a + (1+R)hB)} \\ & + \frac{2\alpha_1 M^{\alpha_3} (((1+R)h - a)B + Ca\phi)AN^{\alpha_2} K^{\alpha_1-1} - 2\delta hB}{(\alpha_3 - 1)B h} \end{aligned}, \quad (12)$$

## 4. Results

This section presents an analysis of the hypothesized economic model developed in this study. The findings are based on a series of simulations that explore different scenarios by varying  $M$  and  $m$  values. We begin by establishing the stability conditions of the model, which can be evaluated as follows:

$$J = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & 0 & a_5 \\ a_6 & a_7 & a_8 & a_9 & 0 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & -\frac{sM}{(\alpha 3 - 1)R} & a_{15} \\ -1 & -1 & a_{16} & a_{17} & 0 & a_{18} \\ 0 & -h & a_{19} & a_{20} & b & -d \\ 0 & 0 & -m & 0 & s & r \end{bmatrix} \quad [1]$$

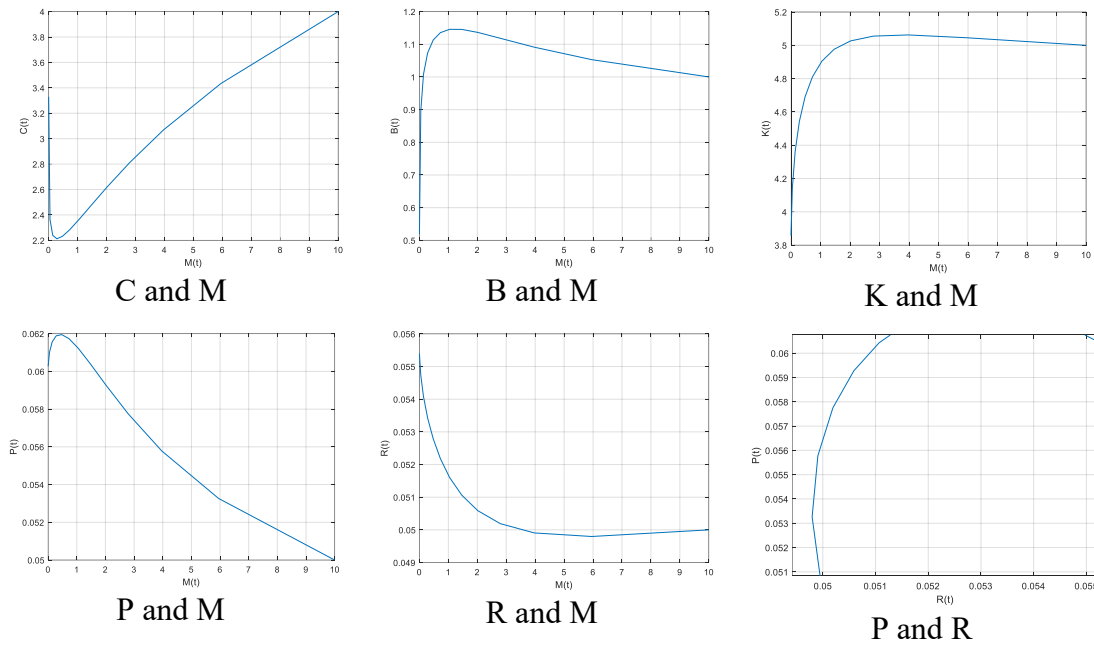
The value of  $a_1$  to  $a_{20}$  are shown in the Appendix.

In this economy, we assume that households have a strong preference for the liquidity services provided by money. They are characterized by a moderate degree of risk aversion, which corresponds to their moderate willingness to substitute consumption across different periods. In addition, production technology features a positive level of total factor productivity, allowing output growth even with fixed inputs. Capital is assumed to be the largest contributor to value addition. The physical capital stock is subject to moderate depreciation. Furthermore, the evolution of the interest rate is equally influenced by price and its feedback. The evolution of the price is most influenced by its feedback. Thus, the values of the parameters assumed in this study are as follows:

$$\begin{aligned} \varphi = 2, \sigma = 1.5, A = 1.5, \alpha_1 = 0.4, \alpha_2 = 0.3, \alpha_3 = 0.3, \delta = 0.07, \\ a = 0.01, b = 0.02, d = 0.01, h = 0.02, s = 0.1, r = 0.1, \rho = 0.1 \end{aligned} \quad [2]$$

which provide that  $\text{Det} = 0.0145$ ,  $\text{Trace} = -8.1830$ , and  $\text{Trace}^2/4 = 16.7404$ , implying stability according to the stability conditions  $\text{Det}(J) > 0$ ,  $\text{Tr}(J) < 0$ , and  $\text{Det}(J) < \text{Tr}^2/4$ .

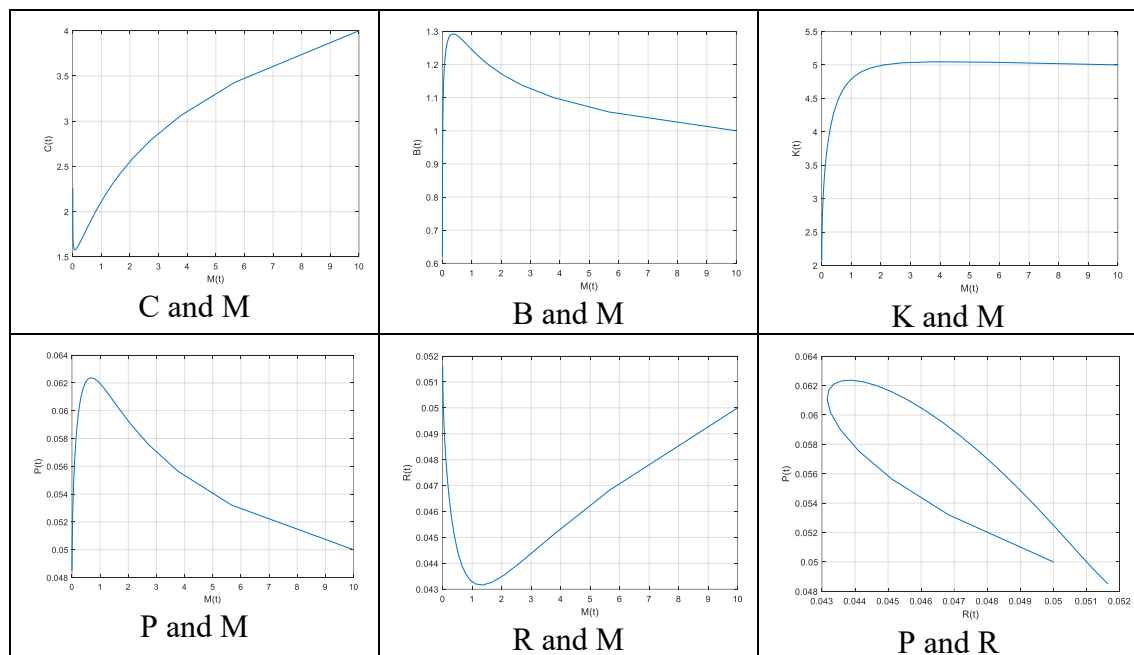
Figure 1:  $M=10, m=0.001$



Source: Author's computation

Figure 1 illustrates the reactions of the variables when  $M=10$  and  $m=0.001$ . Initially, as  $M$  expands within a specific range, 0-1, the variables behave according to conventional economic theories. This injection of  $M$  led to a decrease in  $R$  but an increase in  $C$ ,  $K$ ,  $P$ , and  $B$ . Ultimately, the final graph revealed a negative relationship between  $P$  and  $R$ . However, this dynamic shifts as the  $M$  expansion continues beyond the initial interval.  $R$ , which initially decreased, began to continuously increase. Concurrently, both  $P$  and  $B$  began to gradually decline from their peaks.

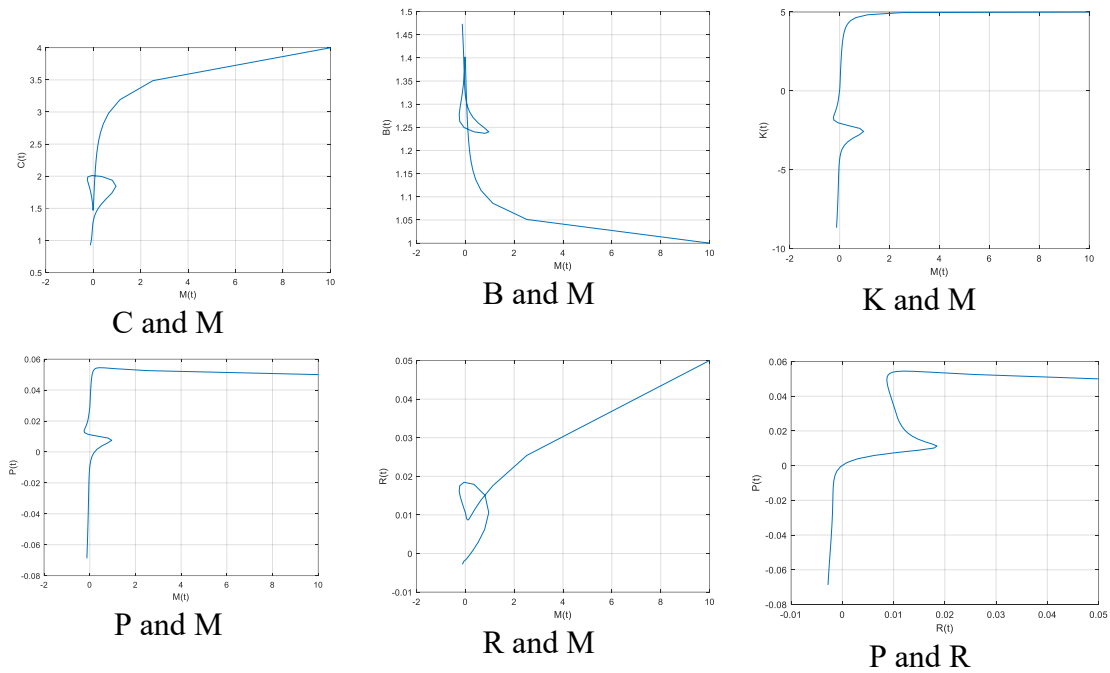
Figure 2:  $M=10, m=0.005$



Source: Author's computation

Figure 2 presents the reactions of the variables when  $M=10$  and  $m=0.005$ . When  $m$  increases, all variables show similar patterns as in Figure 1; that is,  $C$  increases,  $B$  gradually decreases after its peak,  $K$  increases and levels off,  $P$  initially spikes then steadily declines, and  $R$  drops sharply at first, then gradually increases over time. Finally,  $P$  and  $R$  revealed mostly an inverse relationship. However, increasing  $m$  led to the raising of the maximum point and lowering the minimum points of variables.

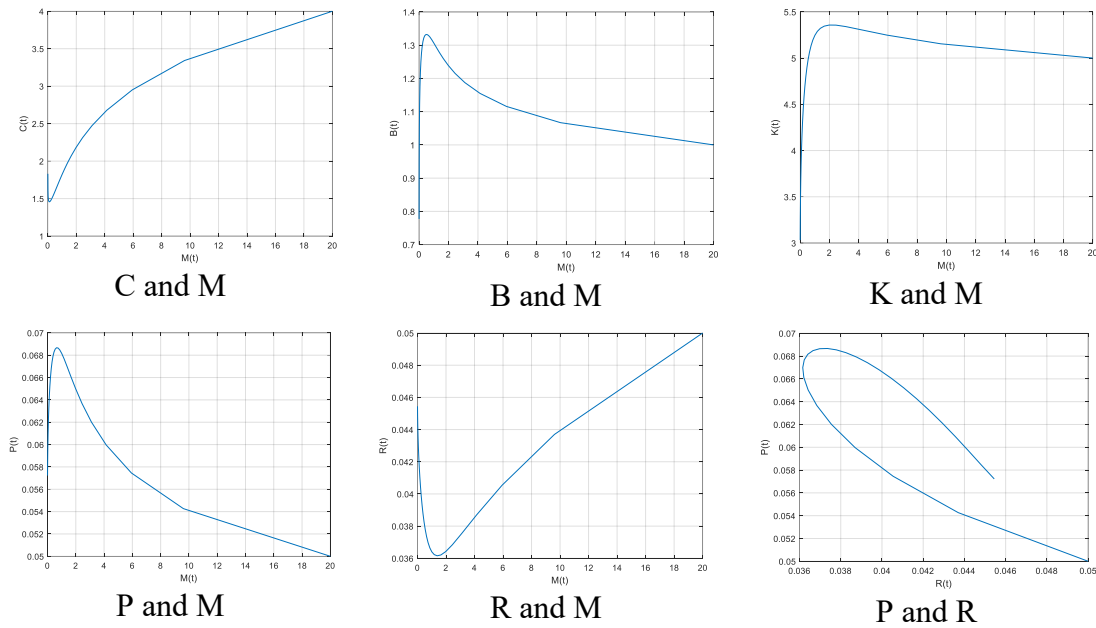
Figure 3:  $M=10$ ,  $m=0.05$



Source: Author's computation

Figure 3, which presents the reactions of key variables when  $M=10$  and  $m=0.05$ , reveals several interesting patterns attributable to the high value of  $m$ .  $C$  initially plummets, then rebounds swiftly and enters a phase of steady growth.  $B$  shows high initial volatility with a sharp spike and drop before gradually decreasing.  $K$  overshoot before the dip.  $P$  demonstrated a swing before rising sharply. Similarly,  $R$  experienced a sharp drop and rebound. The combined graphs of  $P$  and  $R$  revealed a multi-part relationship.

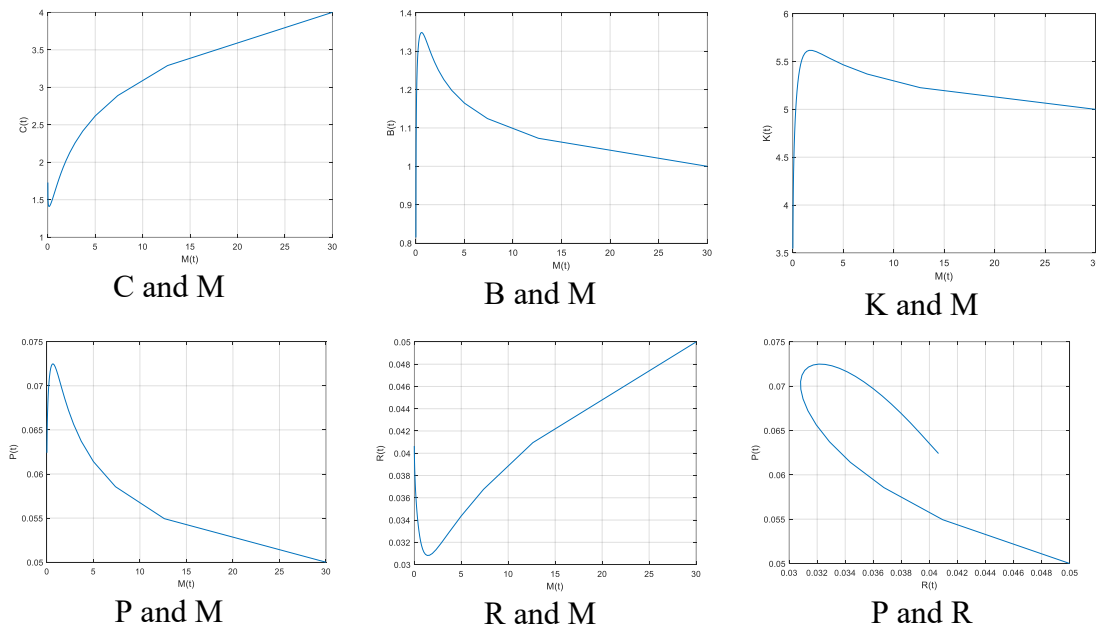
Figure 4:  $M=20$   $m=0.005$



Source: Author's computation

Figure 4, which illustrates the reactions of key variables under the parameters  $M=20$  and  $m=0.005$ , shows that an increase in  $M$  preserves the general dynamic patterns observed in Figure 2. However, a higher value of  $M$  serves to amplify the fluctuations in the variables, leading to higher maximum and minimum points.

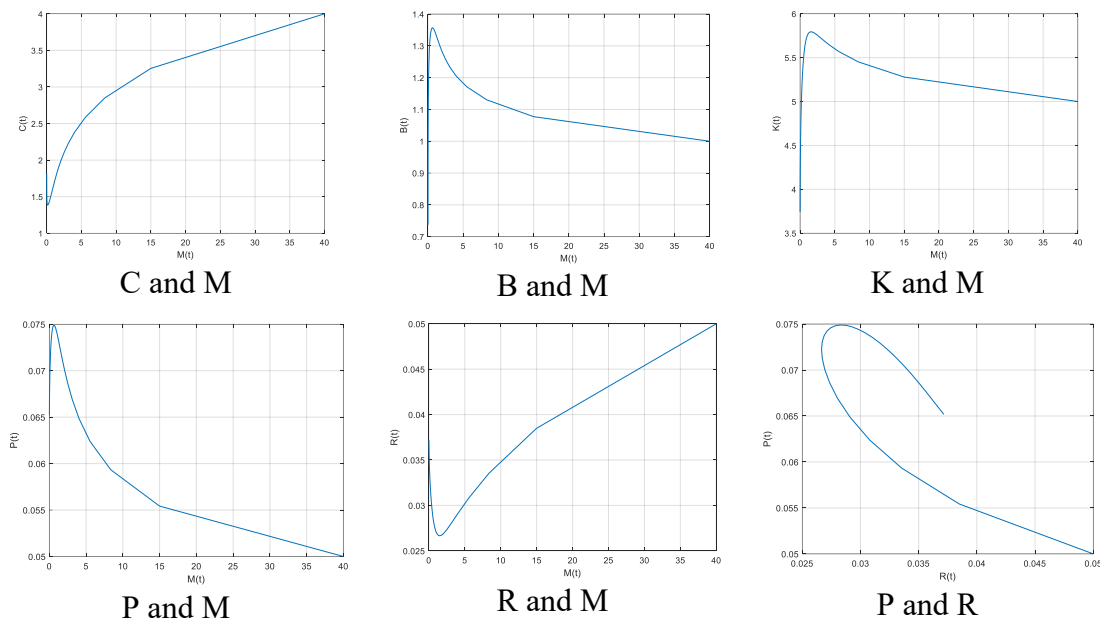
Figure 5:  $M=30$   $m=0.005$



Source: Author's computation

Figure 5 illustrates the dynamic response of the economic variables when  $M = 30$  and  $m = 0.005$ . Consistent with the previous scenario shown in Figure 4, the variables exhibited similar overall patterns of behavior. However, a larger  $M$  amplifies these dynamics, resulting in higher maximum and minimum points. This indicates that a greater  $M$  accelerates the growth of  $C$ ,  $K$ ,  $P$ , and  $B$ , while simultaneously causing  $R$  to decelerate.

Figure 6:  $M=40$   $m=0.005$



Source: Author's computation

Figure 6, which models the economy with parameters  $M=40$  and  $m=0.005$ , shows that the patterns of the economic variables begin to differ from those in Figure 5. A further increase in  $M$  continues to accelerate  $K$  and  $P$  but decelerates  $R$ . However, its effect on  $C$  and  $B$  is no longer precise.

The results in Figures 4, 5, and 6 reveal a clear trend as  $M$  increases from 20 to 40. With each increment, the initial drop in  $R$  became progressively shallower. Simultaneously, the initial levels achieved by  $P$  and  $K$  are consistently higher, indicating a strong positive response to the expanding money supply.

## 5. Discussion

Based on the effects of an increase in  $M$ , the results indicate that, over an initial range of time and monetary expansion,  $C$ ,  $K$ ,  $P$ , and  $B$  all rise, whereas  $R$  declines. However, beyond this initial interval, the behavior of these variables changes. Specifically,  $C$  and  $K$  continued to move in the same direction, maintaining an upward trajectory as  $M$  increased. In contrast,  $B$ ,  $P$ , and  $R$  exhibited a reversal in their paths relative to their initial responses.

The steady rise in  $C$  and  $K$  as  $M$  increases aligns with the fundamental principles of the Quantity Theory of Money (QTM) and Monetarist Theory, which explain the relationship between money supply, inflation, and economic growth. Regarding the successive increase in  $K$ , it offers partial support for the ABCT's concept of malinvestment during periods of monetary expansion (Howden, 2009). These relationships generally support the idea that an increase in the money supply can stimulate economic activity, at least in the short term, as posited by Kemal (2006) and Qayyum (2022).

For the variables that altered their trajectory following the initial interval of increasing  $M$ , the most notable changes were observed in  $B$ ,  $P$ , and  $R$ . In particular,  $B$  began to decline gradually. This pattern is consistent with the findings of Choi and Oh (2000), who argue that a tighter monetary policy raises money demand; by symmetry, a looser policy could be associated with a weakening in money demand. The observed

decline in B may also reflect a saturation point in liquidity preference once the money supply has expanded sufficiently. This outcome stands in tension with the traditional view of money demand, which holds that the public's desired money holdings, motivated by transaction, precautionary, and speculative considerations, are determined primarily by variables such as income, the price level, interest rates, and expectations rather than by the money supply itself. Under this conventional framework, the money supply is typically treated as being exogenous to money demand. Nonetheless, the money supply can indirectly influence money demand by altering the broader macroeconomic environment. Changes in the money supply can affect income, inflation, and interest rates, thereby shaping money demand. For example, an increase in the money supply may reduce interest rates, stimulate spending, and raise income, all of which can increase the demand for transaction balances, and the reverse mechanism may operate when the money supply contracts.

In the case of the inverse relationship between M and P, where P decreases as M rises, observed after passing the first initial interval, it presents an interesting challenge to the straightforward inflation expectations of the QTM and Monetarist Theory. This finding contrasts with the traditional view that an increase in the money supply leads to higher inflation rates (Kemal, 2006; Teles et al., 2016). Instead, it aligns more closely with the Austrian Business Cycle Theory (ABCT), which suggests that monetary interventions can lead to complex and sometimes counterintuitive effects on the economy (Espinosa et al. 2023). However, such an inverse relationship may be explained via QTM, which can be encapsulated in the formula  $P = \frac{MV}{Y}$  that posits a direct relationship where

a larger money supply (M) increases the price level (P), this outcome depends on the assumption of relatively stable velocity (V) and real output (Y). An inverse relationship, where an increase in the money supply corresponds with a fall in prices, is possible but only occurs when other factors forcefully counteract monetary expansion. For instance, if a monetary increase triggers a severe drop in the velocity of money, which may happen during a financial panic when people hoard cash, the overall nominal spending (MV) can fall, pulling prices down. Similarly, if easy credit finances a massive productivity boom, real output (Y) could grow so rapidly that it outpaces the growth in the money supply, leading to deflation. Beyond these mechanics, central banks may expand the money supply in response to falling prices to combat deflation, rather than the money supply causing the price drop. In the case of R, the initial sharp drop in the first interval supports the liquidity effect described by Christiano and Eichenbaum (1992). However, beyond that interval, the positive relationship between M and R supports the findings of Azmy (2014), Dai (2014), Akalpler & Duhok (2018), Mahmood et al. (2017), and Daoud & Al-Ezzi (2023).

Finally, the complex relationship between variables after setting a large value of  $m$  supports the work of Bernanke & Mihov (1996), who state that an excessively high sensitivity ( $\alpha$ ) could lead to a destabilizing policy rate, potentially detrimental to the economy, and harm overall economic performance.

## **6. Policy Implication**

Building on the finding that expansionary monetary policy exerts predictable effects within specific time horizons and money supply ranges, policymakers can design monetary strategies more effectively to achieve defined policy objectives. When the ultimate goal is to foster economic growth, they can seek to identify the optimal degree of monetary stimulus that maximizes growth while minimizing the risk of

macroeconomic instability. This involves avoiding the point at which additional liquidity yields diminishing returns, such that further increases in the money supply no longer generate meaningful gains in economic growth. In this context, policymakers should carefully calibrate both the money supply and the money–interest rate sensitivity parameter to fine-tune policy interventions.

## 7. Future Research

Future research should develop models that can capture more of the state-dependent effects of monetary policy across different scenarios. Researchers should also explore the potential interactions between monetary policy and other economic factors, such as fiscal policy, international trade, and technological innovation.

## 8. Conclusion

This study examined the complex relationships between the money supply ( $M$ ) and key economic variables within a dynamic setting, addressing a significant gap in the monetary economics literature. To achieve this, we developed a model utilizing an optimal control framework and a Hamiltonian approach. The model uniquely incorporates money as both a factor in the Cobb-Douglas production function, alongside capital ( $K$ ) and labor ( $N$ ), and as a component of the utility function. This approach allowed for the analysis of the dynamic interactions among  $M$ , consumption ( $C$ ), money demand ( $B$ ),  $K$ , price ( $P$ ), and the interest rate ( $R$ ). The simulation results reveal interesting non-linear relationships that both support and challenge established economic theories. We found that expansionary monetary policy generally accelerates  $C$ ,  $K$ ,  $P$ , and  $B$ . Specifically, the results indicate a positive relationship between  $M$  and  $K$ , confirming that monetary policy measures can stimulate economic growth. However, the initial sharp drop in  $R$  supported the theoretical liquidity effect, but this relationship shifted beyond the initial monetary injection, where a positive correlation between  $M$  and  $R$  was observed. Furthermore, the findings present significant challenges to traditional expectations, i.e., an inverse relationship between  $M$  and  $P$  was observed after the initial interval, contradicting the straightforward inflation expectations of the Quantity Theory of Money (QTM) and Monetarist Theory. This inverse relationship aligns more closely with the concepts of the Austrian Business Cycle Theory (ABCT). Crucially, the analysis highlights the sensitivity of the economic system to the money-interest rate sensitivity parameter ( $m$ ), which captures the effect size of the money supply on interest rates. The results revealed that setting  $m$  to a high value induced volatile and destabilizing dynamics for all variables. These findings indicate that while expansionary monetary policy can stimulate growth, policymakers must exercise precision in calibrating the amount of  $M$  and  $m$  to identify the optimal level of monetary stimulus that maximizes  $K$ , which is a proxy for economic growth, while avoiding economic instability.

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## References

- Abdul Qayyum, A. (2022). Money, inflation, and growth in Pakistan. *The Pakistan Development Review*, 45(2), 203–212.
- Adjemian, M. K., Salin, D., Meyer, S., & Arita, S. (2023). Factors affecting recent food price inflation in the United States. *Applied Economic Perspectives and Policy*, 46(2), 648–676.
- Akalpler, E., & Duhok, D. (2018). Does monetary policy affect economic growth: Evidence from Malaysia. *Journal of Economic and Administrative Sciences*, 34(1), 2–20.
- Alvarez, F., Weber, W. E., & Lucas, R. E. (2001). Interest rates and inflation. *American Economic Review*, 91(2), 219–225.
- Andersen, L. C., & Carlson, K. M. (1970). A monetarist model for economic stabilization. *Federal Reserve Bank of St. Louis Review*, October, 45–66.
- Azmy, A. (2014). Evaluasi Kebijakan Pemerintah Dalam Menjaga Stabilitas Pertumbuhan Ekonomi Tahun 2013 [Government policy evaluation in maintaining the stability of economic growth in 2013]. *Signifikan: Jurnal Ilmu Ekonomi*, 3(1), 95–112.
- Benchimol, J. (2014). Money in the production function: A new Keynesian DSGE perspective. *Southern Economic Journal*, 82(1), 152–184.
- Benchimol, J., & Fourçans, A. (2011). Money and risk in a DSGE framework: A Bayesian application to the Eurozone. *Journal of Macroeconomics*, 34(1), 95–111.
- Bernanke, B. S., & Mihov, I. (1996). *What does the Bundesbank target?* (Working Paper No. 5764). Cambridge, MA: National Bureau of Economic Research.
- Brzezinski, A., Chen, Y., Palma, N., & Ward, F. (2022). *The vagaries of the sea: Evidence on the real effects of money from maritime disasters in the Spanish Empire* (Economics Discussion Paper Series No. EDP-1906). School of Social Sciences, The University of Manchester.
- Buthlezi, E. M. (2023). Impact of money supply in different states of inflation and economic growth in South Africa. *Economies*, 11(2).
- Chindengwike, J. (2022). Nexus between money supply and economic development in East Africa countries: An empirical study using ARDL. *Journal of Global Economy*, 18(4), 237–250.
- Choi, W. G., & Oh, S. (2000). *Endogenous money supply and money demand* (IMF Working Paper No. WP/00/188). International Monetary Fund. Retrieved from <https://ssrn.com/abstract=1445697>
- Christiano, L. J., & Eichenbaum, M. (1992). *Liquidity effects, monetary policy and the business cycle* (Working Paper Series No. WP-92-15). Federal Reserve Bank of Chicago. Retrieved from [https://fraser.stlouisfed.org/files/docs/historical/frbchi/workingpapers/frbchi\\_workingpaper\\_1992-15.pdf](https://fraser.stlouisfed.org/files/docs/historical/frbchi/workingpapers/frbchi_workingpaper_1992-15.pdf)
- Dai, S.-C. (2014). Funding liquidity and money market interest rate—New evidence from China. *2014 International Conference on Management Science & Engineering 21th Annual Conference Proceedings*, 1147–1156.
- Daoud, H. E. D., & Al-Ezzi, R. (2023). The cointegration relationship between money supply, exchange rate and economic growth in Iraq (2004–2020). *Business Series*, 2(1), 9–35.
- De Long, J. B. (2000). The triumph of monetarism?. *Journal of Economic Perspectives*, 14(1), 83–94.

- Espinosa, V. I., Huerta de Soto, J., & Alonso-Neira, M. A. (2023). The central banking system paradox. *Economics & Sociology*, 16(4), 56–72.
- Galí, J. (2018). The state of new Keynesian economics: A partial assessment. *Journal of Economic Perspectives*, 32(3), 87–112.
- Gavin, W. T., & Kydland, F. E. (1997). *Endogenous money supply and the business cycle* (Working Paper No. 1995-010D). Federal Reserve Bank of St. Louis. Retrieved from <https://fraser.stlouisfed.org/title/working-papers-federal-reserve-bank-st-louis-9029/endogenous-money-supply-business-cycle-715990>
- Geromichalos, A., Licari, J. M., & Suárez-Lledó, J. (2007). Monetary policy and asset prices. *Review of Economic Dynamics*, 10(4), 761–779.
- Howden, D. (2009). Knowledge shifts and the business cycle: When boom turns to bust. *The Review of Austrian Economics*, 23(2), 165–182.
- Huerta De Soto, J., Bagus, P., & Sánchez-Bayón, A. (2021). Principles of monetary & financial sustainability and wellbeing in a post-COVID-19 world: The crisis and its management. *Sustainability*, 13(9).
- Ivanov, M., Petkovski, M., & Naumovska, E. (2014). Money-in-the-utility-function: Model simulations and money demand estimation in the case of the Republic of Macedonia. *South East European Journal of Economics and Business*, 9(2), 7–20.
- Jones, L. E., & Manuelli, R. E. (1995). Growth and the effects of inflation. *Journal of Economic Dynamics and Control*, 19(8), 1405–1428.
- Keister, T., Martin, A., & McAndrews, J. (2008). Divorcing money from monetary policy. *SSRN Electronic Journal*, 14(2), 41–56.
- Kemal, M. A. (2006). Is inflation in Pakistan a monetary phenomenon?. *The Pakistan Development Review*, 45(2), 213–220.
- Liu, L., Jain, V., Ramos-Meza, C. S., Shabbir, M. S., Salman, A., Abdalla, A. A., & Bashir, T. (2022). Can money supply endogeneity influence bank stock returns? A case study of South Asian economies. *Environment, Development and Sustainability*. 26, 2775–2787.
- Madurapperuma, W. (2023). Money supply, inflation and economic growth of Sri Lanka: Co-integration and causality analysis. *Journal of Money and Business*, 3(2), 227–236.
- Mahmood, H., Khalid, S., & Waheed, A. (2017). The impact of monetary strategies on economic growth: An empirical analysis for Pakistan. *Asian Journal of Empirical Research*, 7(10), 260–268.
- Mankiw, N. G. (1992). The reincarnation of Keynesian economics. *European Economic Review*, 36(2–3), 559–565.
- Oyadeyi, O. O. (2024). Financial development, monetary policy, and the monetary transmission mechanism—An asymmetric ARDL analysis. *Economies*, 12(8).
- Oyadeyi, O. O., Oyadeyi, O. A., & Iyoha, F. A. (2024). Exchange rate pass-through on prices in Nigeria—A threshold analysis. *International Journal of Financial Studies*, 12(4).
- Palley, T. (2023). Keynes’ denial of conflict: Why the general theory is a misleading guide to capitalism and stagnation. *The Japanese Political Economy*, 49(1), 7–34.
- Poterba, J. M., & Rotemberg, J. J. (1986). *Money in the utility function: An empirical implementation* (Working Paper No. 1796). National Bureau of Economic Research.
- Qayyum, A. (2022). Money, inflation, and growth in Pakistan. *The Pakistan Development Review*, 45(2), 203–212.
- Satti, A. U. H., Malik, W. S., & Saghir, G. (2007). New Keynesian Phillips curve for Pakistan. *The Pakistan Development Review*, 46(4II), 395–404.

- Shahid Ali, S., & Somia Irum, S. (2022). Whether fiscal stance or monetary policy is effective for economic growth in case of South Asian countries? *The Pakistan Development Review*, 47(4II), 791–799.
- Sidrauski, M. (1967). Rational choice and patterns of growth in a monetary economy. *The American Economic Review*, 57(2), 534–54
- Sinai, A., & Stokes, H. H. (1972). Real money balances: An omitted variable from the production function?. *The Review of Economics and Statistics*, 54(3), 290-296.
- Smith, B. D. (1985). American colonial monetary regimes: The failure of the quantity theory and some evidence in favour of an alternate view. *The Canadian Journal of Economics*, 18(3), 531-565.
- Sumi, M. S. S., & Rabeta, M. (2024). Monetary policy and stock market dynamics: An empirical perspective. *International Journal of Innovative Science and Research Technology*, 9(8), 586–592.
- Teles, P., Valle E Azevedo, J., & Uhlig, H. (2016). Is quantity theory still alive?. *The Economic Journal*, 126(591), 442–464.
- Urich, T., & Wachtel, P. (1984). The effects of inflation and money supply announcements on interest rates. *The Journal of Finance*, 39(4), 1177–1188.
- White, L. H. (2008). Did Hayek and Robbins deepen the great depression?. *Journal of Money, Credit and Banking*, 40(4), 751–768.

## Appendix

### Elements of the Jacobian Matrix (J)

(This is just an example. Please use the name you see fit)

$$\begin{aligned}
 a_1 &= \frac{1}{B(C\phi - B)^2 m(\sigma^2 \phi - 2(-1 + \sigma)^2)h} (-M^{\alpha^3} A(((1 + R)h - a)B + 2Ca\phi) \\
 &\quad (C\phi - B)^2 m\alpha 1(\sigma\phi + \sigma - 1)N^{\alpha^2} K^{\alpha^{1-1}} - B(A\alpha 3(((1 + R)h - a)B^2 + 2BCa\phi - C^2 a\phi^2) \\
 &\quad N^{\alpha^2} K^{\alpha^1} (-1 + \sigma)sM^{\alpha^{3-1}} + (C\phi - B)^2 m(\sigma(\rho - \delta)\phi + 2(\rho - \frac{1}{2}\delta - \frac{1}{2}b)(-1 + \sigma))h)) \\
 a_2 &= \frac{AN^{\alpha^2} \phi(m\alpha 1M^{\alpha^3} a(C\phi - B)^2 (\sigma\phi + \sigma - 1)K^{\alpha^{1-1}} + B^2 K^{\alpha^1} M^{\alpha^{3-1}} \alpha 3h^2 s(1 + R)(-1 + \sigma))C^2}{(\sigma^2 \phi - 2(-1 + \sigma)^2)B^2 (C\phi - B)^2 hm} \\
 &\quad A(((1 + R)h - a)B + Ca\phi)N^{\alpha^2} C((\sigma\phi + \sigma - 1)(C\phi - B)M^{\alpha^{3-1}} \\
 a_3 &= -\frac{m\alpha 1K^{\alpha^{1-1}} - BhK^{\alpha^1} sM^{\alpha^{3-2}} (\alpha 3 - 1)(-1 + \sigma))\alpha 3}{(\sigma^2 \phi - 2(-1 + \sigma)^2)B(C\phi - B)hm} \\
 &\quad A(((1 + R)h - a)B + Ca\phi)(-M^{\alpha^3} m(C\phi - B)(\alpha 1 - 1)(\sigma\phi + \sigma - 1)K^{\alpha^{1-2}} \\
 &\quad + BhK^{\alpha^{1-1}} s\alpha 3M^{\alpha^{3-1}} (-1 + \sigma))N^{\alpha^2} C\alpha 1 \\
 a_4 &= \frac{(\sigma^2 \phi - 2(-1 + \sigma)^2)B(C\phi - B)hm}{(\sigma^2 \phi - 2(-1 + \sigma)^2)B(C\phi - B)hm} \\
 a_5 &= \frac{N^{\alpha^2} AC(-M^{\alpha^3} \alpha 1m(C\phi - B)(\sigma\phi + \sigma - 1)K^{\alpha^{1-1}} + BK^{\alpha^1} M^{\alpha^{3-1}} \alpha 3hs(-1 + \sigma))}{m(C\phi - B)(\sigma^2 \phi - 2(-1 + \sigma)^2)} \\
 a_6 &= \frac{AN^{\alpha^2} (3(\sigma - \frac{2}{3})a(C\phi - B)^2 M^{\alpha^3} m\alpha 1K^{\alpha^{1-1}} + B^2 K^{\alpha^1} M^{\alpha^{3-1}} \alpha 3h^2 s\sigma(1 + R))}{(-2 + (\phi - 2)\sigma^2 + 4\sigma)(C\phi - B)^2 hm} \\
 &\quad 3((\sigma - \frac{2}{3})((1 + R)h - a)M^{\alpha^3} A(C\phi - B)^2 m\alpha 1N^{\alpha^2} K^{\alpha^{1-1}} \\
 &\quad - 2(A((C\phi - \frac{1}{2}B)B(1 + R)h + \frac{1}{2}a(C\phi - B)^2)\alpha 3N^{\alpha^2} K^{\alpha^1} s\sigma M^{\alpha^{3-1}} \\
 &\quad - 2(\rho - \frac{3}{4}\delta - \frac{b}{4})\sigma - \frac{1}{2}\rho + \frac{1}{2}\delta)(C\phi - B)^2 m)h) \\
 a_7 &= \frac{\phi(C\phi - B)^2 m(\sigma^2 \phi - 2(-1 + \sigma)^2)h}{\phi(C\phi - B)^2 m(\sigma^2 \phi - 2(-1 + \sigma)^2)h} \\
 a_8 &= \frac{3((\sigma - \frac{2}{3})M^{\alpha^{3-1}} (C\phi - B)m\alpha 1K^{\alpha^{1-1}} - \frac{BK^{\alpha^1} M^{\alpha^{3-2}} hs\sigma(\alpha 3 - 1)}{3})A(((1 + R)h - a)B + Ca\phi)N^{\alpha^2} \alpha 3}{(-2 + (\phi - 2)\sigma^2 + 4\sigma)(C\phi - B)\phi hm} \\
 a_9 &= -\frac{A(((1 + R)h - a)B + Ca\phi)N^{\alpha^2} (-3(\sigma - \frac{2}{3})M^{\alpha^3} (\alpha 1 - 1)(C\phi - B)mK^{\alpha^{1-2}} + BK^{\alpha^{1-1}} \alpha 3hs\sigma M^{\alpha^{3-1}})\alpha 1}{(\sigma^2 \phi - 2(-1 + \sigma)^2)(C\phi - B)\phi hm} \\
 a_{10} &= -\frac{N^{\alpha^2} AB(-3m\alpha 1(C\phi - B)M^{\alpha^3} (\sigma - \frac{2}{3})K^{\alpha^{1-1}} + BK^{\alpha^1} M^{\alpha^{3-1}} \alpha 3hs\sigma)}{m\phi(C\phi - B)(\sigma^2 \phi - 2(-1 + \sigma)^2)} \\
 a_{11} &= -\frac{1}{(\alpha 3 - 1)B((1 + R)h - a)B + Ca\phi)^2 (C\phi - B)^2 \alpha 3hAK} (M(-h^2 m^2 \phi B^2 K^{-\alpha^1} N^{-\alpha^2} K(C\phi - B)^2 (1 + R)M^{-\alpha^{3+1}} \\
 &\quad + A(-2(((1 + R)h - a)B + Ca\phi)^2 a\alpha 3N^{\alpha^2} AK\phi M^{\alpha^3} (C\phi - B)^2 m\alpha 1K^{\alpha^{1-1}} \\
 &\quad + Bh(((1 + R)h - a)B + Ca\phi)^2 \alpha 3^2 N^{\alpha^2} AK^{\alpha^1} BshK\phi(1 + R)M^{\alpha^{3-1}} \\
 &\quad - (C\phi - B)^2 m(((1 + R)h - a)B + Ca\phi)^2 \alpha 1\alpha 3 + hm\phi BKM\alpha))M)
 \end{aligned}$$

$$a_{12} = -\frac{1}{(\alpha 3 - 1)B((1 + R)h - a)B + Ca\phi)^2(C\phi - B)^2\alpha 3hA} (M(\phi M^{\alpha 3}(((1 + R)h - a)B + Ca\phi)^2Ca\alpha 3AK^{\alpha 1}N^{\alpha 2} \\ Bh(((1 + R)h - a)B + Ca\phi)^2\alpha 3^2N^{\alpha 2}AK^{\alpha 1}CshK\phi(1 + R)M^{\alpha 3-1} - 2aM^{\alpha 3}A\phi(C\phi - B)^2 \\ (((1 + R)h - a)B + Ca\phi)^2K\alpha 1\alpha 3N^{\alpha 2}CmK^{\alpha 1-1} + B^2h(((1 + R)h - a)B + Ca\phi)^2\alpha 3^2N^{\alpha 2}AK^{\alpha 1}CshK\phi(1 + R)M^{\alpha 3-1} \\ + (C\phi - B)^2m(((1 + R)h - a)B + Ca\phi)^2\alpha 1\alpha 3 - hm\phi CKMa)))m \\ + \frac{2}{2})$$

$$a_{13} = \frac{1}{((1 + R)h - a)B + Ca\phi)(-C\phi + B)Bh\alpha 3(\alpha 3 - 1)RAK m} (-dhm^2BN^{-\alpha 2}M^{-\alpha 3+1}K^2(C\phi - B)^2K^{-\alpha 1-1} \\ + A(2(\alpha 1 - 1)AM^{\alpha 3}mK^2(((1 + R)h - a)B + Ca\phi)N^{\alpha 2}(C\phi - B)K^{\alpha 1-2} \\ + B(AKN^{\alpha 2}(K\alpha 3(((1 + R)h - a)B + Ca\phi)sM^{\alpha 3-1} - m\alpha 1M^{\alpha 3}(1 + R)(C\phi - B)))K^{\alpha 1-1} \\ + m((1 + R)AK^{\alpha 1}N^{\alpha 2}M^{\alpha 3} - B - C)(C\phi - B)h)\alpha 3(((1 + R)h - a)B + Ca\phi)))M$$

$$a_{14} = \frac{1}{(\alpha 3 - 1)((1 + R)h - a)B + Ca\phi)(C\phi - B)h\alpha 3AK m} (-M(2K^{\alpha 1}(\alpha 1 - 1)(C\phi - B)M^{\alpha 3}mA^2K^{-\alpha 1+1} \\ + A(\phi M^{\alpha 3}(((1 + R)h - 2a)B + 2Ca\phi)AK^{\alpha 1}N^{\alpha 2}R(-C\phi + B)(\alpha 3 - 2)A + 2M^{\alpha 3}(((1 + R)h - a)B + Ca\phi) \\ + B(N^{\alpha 2}RAK^{\alpha 1}A^2\alpha 3^2K(((1 + R)h - a)B + Ca\phi)^2sM^{\alpha 3-1}))(-C\phi + B)(-dmM^{-\alpha 3}N^{-\alpha 2}KM(\alpha 3 - 1)K^{-\alpha 1} \\ + A(\alpha 1RK^{\alpha 1}A\alpha 3(1 + R)M^{\alpha 3}(((1 + R)h - a)B + Ca\phi)N^{\alpha 2} - \alpha 1R((1 + R)h - a)\alpha 3B^2 \\ + ((-(1 + R)((\alpha 1\delta + b - 2\delta - 2r)K + \alpha 1C)R + K(2Mm - Ps))\alpha 3 + 2mKM)R)h \\ + (((\alpha 1\delta + b - 2\delta - 2r)K - \alpha 1C(\phi - 1))R + K(2Mm - Ps))\alpha 3)a)B - \alpha 3((\alpha 1\delta + b - 2\delta - 2r)K \\ + \alpha 1C)Ra\phi C)m)))M)$$

$$a_{15} = \left( \begin{aligned} & \frac{\alpha 1M^{\alpha 3}AN^{\alpha 2}K^{\alpha 1-1}}{\alpha 3 - 1} - \frac{sK^{\alpha 1}\alpha 3AN^{\alpha 2}hBM^{\alpha 3-1}}{(\alpha 3 - 1)m(-C\phi + B)} \\ & - \frac{r}{\alpha 3 - 1)R} - \frac{Mm - Ps - Rr}{(\alpha 3 - 1)R^2} - \frac{rhB}{(\alpha 3 - 1)(BRh + Ca\phi - Ba + Bh)} \\ & \frac{(-dmK^{-\alpha 1}M^{-\alpha 3+1}N^{-\alpha 2}(-C\phi + B))}{(\alpha 3 - 1)\alpha 3A(BRh + Ca\phi - Ba + Bh)^2} \end{aligned} \right) M$$

$$a_{16} = (1 + R)AK^{\alpha 1}N^{\alpha 2}M^{\alpha 3-1}\alpha 3$$

$$a_{17} = (1 + R)AK^{\alpha 1-1}\alpha 1N^{\alpha 2}M^{\alpha 3} - \delta$$

$$a_{18} = AK^{\alpha 1}N^{\alpha 2}M^{\alpha 3}$$

$$a_{19} = aAK^{\alpha 1}N^{\alpha 2}M^{\alpha 3-1}\alpha 3$$

$$a_{20} = \alpha AK^{\alpha 1-1}\alpha_1N^{\alpha 2}M^{\alpha 3}$$