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Examine First Grade Students' Strategies of Solving Open-ended Problems on Addition

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Abstract. The study aimed to examine the first Grade students' strategies of solving open-ended problems on addition. Methodology regarded interpretive paradigm. Participants included 76 grade 1 students of the academic year 2019 in Khon Kaen University Demonstration Primary School, Khon Kaen, Thailand. The addition learning activities were organized through open approach which immersed in lesson study Professional Learning Community. The intervention of addition was provided to enhance students to learn various ways of solving open ended problems about addition. Data about students' strategies of solving open ended problems was collected through participant observation, interview, lesson study reflection, and students' tasks. Students' strategies of solving open ended problems, then, was interpreted and categorized into groups. The findings revealed (1) students' strategies of solving open ended problems on addition, and (2) suggested about student fundamental strategies of solving problem on addition. Students' strategies of solving open ended problems on addition could be categorized into 6 strategies. These included 1) total counting, 2) counting, 3) making ten full, 4) separation option, 5) separation plus addition and 6) separation plus addition and separation. The reflection of lesson study suggested some fundamental strategies of solving problem on addition. These included 1) practicing about making ten full by using blocks and rail blocks, 2) practicing about using blocks and symbols, and 3) teachers' providing of encourage questions to find other ways of addition.

Keywords: Addition, Strategies of solving problems, Open approach, Lesson study

1. Introduction

Thailand education still keep going on attempting of moving traditional teaching and learning into regarding constructivism. Previous Thailand educational reform mentioned constructivist paradigm of learning, however, the majority of teaching and learning could not building up school culture for constructivist teaching and learning. Expected learning activities considered leaerner centred which should be organized with regard to individual differences and relevant to their learning environment in everyday life (Yuenyong and Thathong, 2015). Constructivist learning is to let students to find personal relevance in their studies, share control over their learning, be confident to express questions or concerns about their learning, be aware that nature of disciplines (e.g. mathematics), and interact with others when conception is needed to develop. Quality of classroom environment has a significant impact on students' learning (Taylor et.al., 1997; Thao-Do et.al., 2016; Tupsai et.al., 2015; Udomkan et.al., 2015).

mathematics open-ended problems probably provide some constructivist learning environment.

The mathematics open-ended problems would enhance student divergent thinking. The mathematics open-ended problems provided a constructive mathematics education that should provide challenging tasks and an environment in order to allow students to construct their own concrete and informal problem solution strategies. The implementing of open-ended problems allows students to explore experientially real and mathematically real context problems. It is another issue of provide learning environment for constructing mathematical knowledge (Gravemeijer and Doormann, 1999; Pehkonen, 1995; Pehkonen, 1997; Woranetsudathip and Yuenyong, 2015).

The open-ended problems were widely used as mathematics investigations. However, open-ended problems were implemented in different names; for examples, in the Netherlands called "realistic mathematics", but in Japan called “open approach” (Clarke and Sullivan, 1992, Kadroon & Inprasiha, 2011; Mason 1991, Nohda, 1991; Wiliam, 1994; Woranetsudathip et.al., 2021; Zimmermann, 1991). Regarding on internationally implementing of the open-ended problems, there were two fundamental consensus views. These included (1) the open-ended problems should be provided as familiar and interesting subjects in order to fit to every student experiences, and (2) it should be provided in diverse levels where fit to many possible solutions (Nohda, 1995, 2000). The open approach was also widely implemented in Thailand mathematics classroom after President of Thailand society of Mathematics Education Prof Maitree Inprasitha introduced it in Thailand since 2002. The open approach was provided in four phases. These included 1) posing open- ended problem; 2) students’ self-learning; 3) whole class discussion and comparison; and 4) summarization through connecting students’ mathematical ideas emerged in the classroom (Kadroon & Inprasiha, 2013; Woranetsudathip et.al., 2021).

To implement open approach in the first Grade students, students’ prior knowledge could be taken into account for open approach mathematics lesson plan. In general, the first Grade students study school mathematics about numbers, addition and subtraction. Natural numbers from 1 to 10 taught is an finite aggregate, and teachers help students get acquainted with relational operator (“less than”, “greater than” or “equal”) by using one to one correspondence. The concept of addition and subtraction could be linked with what we could make sense in their daily life (Nguyen, 2018; Woranetsudathip et.al., 2021). It is quite new and challenge for us to enhance Thailand Grade 1 Thai students’ learning about mathematical ideas on addition through open approach. It may have some difficulties for a mathematics teacher to develop some open-ended problems for open approach. In order to develop open-ended problems that fit to every student experiences and fit to various possible solutions, teachers working together through lesson study may find the innovative lesson plan of open approach on addition (Woranetsudathip et.al., 2021).

Lesson study could enhance teachers to improve mathematics teaching through open approach. The lesson study may allow us to design and delivery of one mathematics lesson of mathematical ideas on addition through open approach (Treffers, 1991). A lesson study is collaborative planning of a research lesson. It should involves small groups around 4 – 6 teachers who teach students in same level and/or content. When the planning begins they need to design the lesson by concerning on the goals of improving student learning outcomes. After the lesson is developed, a teacher from group will teach the lesson. And, the rest of group members will observe and collect data about the lesson process. The data collection may focus on students’ learning for the specific topic taught, and the motivation, behaviors, attitudes and responses of the students towards learning. These varieties of sources and data were interpreted to reflect to the group as individual reflections. Regarding to these reflection, the team need to review and revise the lesson

for the next teaching. Another team member then teaches the revised lesson to another class of pupils. Once again, observations are made and data gathered for further refinement (Lewis and Hurd, 2011; Matoba, 2005; Stigler and Hiebert 1999; Woranetsudathip et.al., 2021).

This study would like to enhance students to learn addition through open approach which the learning activities should be developed in the Professional Learning Community (PLC) of lesson study. The study, therefore, aimed to examine the first Grade students' strategies of solving open-ended problems on addition. This study will adopted the Inprasitha (2010) process of lesson study to implement in the mathematics classroom. These process consist of 3 phases including 1) collaboratively design a research lesson 2) collaboratively observe the research lesson and 3) collaboratively discuss and reflect on the research lesson, in order to help teachers to get a new vision on their classroom teaching practice.

2. Methodology

Methodology regarded interpretive paradigm. First Grade students' strategies of solving opened problems on addition were examined through students' tasks. The addition learning activities were organized through open approach which immersed in the environment of lesson study. The context of classroom, therefore, supported teachers to work collaboratively in planning, observing and reflecting under process of lesson study in order to understand what students developed the strategies of solving open problems on addition.

2.1 Participants

Participants included 76 grade 1 students of the academic year 2019 in Khon Kaen University Demonstration Primary School, Khon Kaen, Thailand.

2.2 Intervention of addition learning activities through open approach in context of lesson study

Intervention was about the learning activities of the unit 2 of addition. Previously, the unit 1 of addition provided the definition of addition as adding up and counting up. Regarding on the unit 1, the learning activities of the unit 2 of addition then enhanced students to learn various ways of solving open ended problems about addition.

The unit 2 lesson plans of addition learning activities were developed through open approach. The learning activities were developed with cooperation among researchers and teachers in Professional Learning Community (PLC) of lesson study. The lesson study process included creating a learning management plan, observing teaching, and reflecting lesson results after teaching. PLC of lesson study, therefore, focused on developing about addition learning activities through open approach. Based on steps of open approach, the addition learning activities through open approach were provided based on four phases, including 1) posing an open-ended problem; 2) students' self-learning; 3) whole class discussion and comparison; and 4) summarization through connecting students' mathematical ideas emerged in the classroom. Finally, the 4 lesson plans were provided. These included lesson plans of (1) How to, (2) lesson plan about 8 + 3, (3) lesson plan about 3 + 9, and (4) lesson plan about 8 + 6. Lesson plan 1 (how to) is the learning activity about using blocks to represent and connection definition of addition. Students have learned already about definition of addition in the previous unit of addition. Lesson plan 2 was about the learning activity for some solving strategies of addition in the open ended problem of 8+3. Lesson plan 3 was about the learning activity for some solving strategies of addition in the open ended problem of 3+9. And, lesson plan 4 was about the learning activity for some solving strategies of addition in the open ended problem of 8+6.

Regarding on lesson study, during the learning and teaching activities, the fellow teachers observed the class and recorded the class environment through notes and video recording. After that, all teachers met again for making an analysis criticism and evaluation of the lesson plan in order to examine the appropriateness of the teacher's performance, materials used, the students' performance, and the problems occurred in the classroom. Finally, the teachers discussed and revised the lesson plans based on their observations and reflections.

2.3 Data collection and analysis

Data about students' strategies of solving open ended problems was collected through participant observation, interview, lesson study reflection, and students' tasks. Some suggested ideas from participant observation, interview, and lesson study reflection supported the researchers to understand students' ideas about strategies of solving open ended problems. Students' strategies of solving open ended problems, then, was interpreted and categorized into groups.

3. Findings and discussion

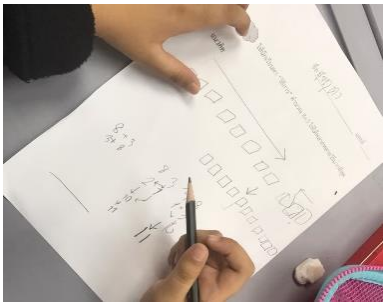
It found that the intervention of addition learning activities through open approach in context of lesson study enhanced the first Grade students to develop various strategies of solving open ended problems on addition. The findings will clarify (1) students' strategies of solving open ended problems on addition, and (2) suggested about student fundamental strategies of solving problem on addition.

3.1 Students' strategies of solving open ended problems on addition

Students' strategies of solving open ended problems on addition could be categorized into 6 strategies. These included 1) total counting, 2) counting, 3) making ten full, 4) separation option, 5) separation plus addition and 6) separation plus addition and separation.

1) Total counting

Addition solving problem about total counting of 8+3, students had to provide eight blocks and then provide another 3 blocks as the picture 1. Then they counted the first group of block by counting adding one by one (from one to eight). Then, they counted the second group of block by keep counting from the first group of block (from nine to eleven).



Picture 1: Students' total counting

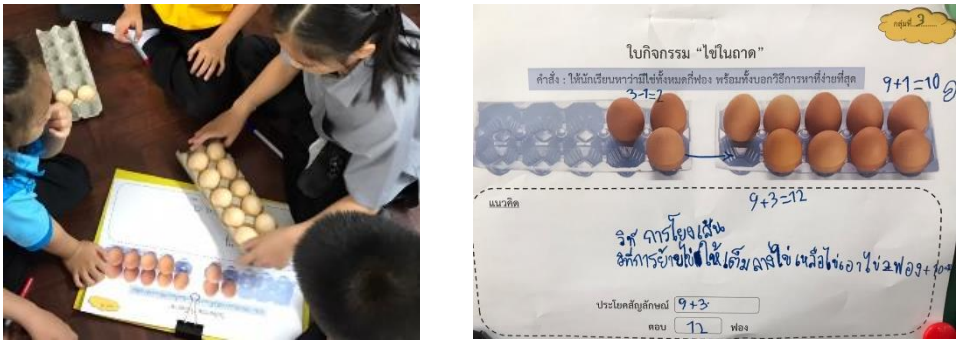
2) Counting

Students counted the picture of monkeys. They counted each of pictures by different ways of counting (e.g. counting two by two, counting three and so on). In the picture 2, students circled at two of monkeys in order to represent their counting two and then keep counting.

Picture 2: Students' counting

3) Making ten full

Making a full ten, students will combine the two numbers into ten. Or add items to be added or combined with another group to a full tenth. Students may keep number sense about block of ten in their mind. And then, they have to put the number to full the block of ten. The picture 3 showed what students solved problem on addition of situation of 3+9. See picture 3 (b), the left block of ten has three eggs and the right block has nine eggs. Students, therefore, need to full the right block of ten. So, they need to move one egg of the right block of ten to the left block. So, the right block has full ten. The left block remains two eggs. Then, the situation of 3+9 could be solved as 2+10.



(a) (b)

Picture 3: Students' making ten full

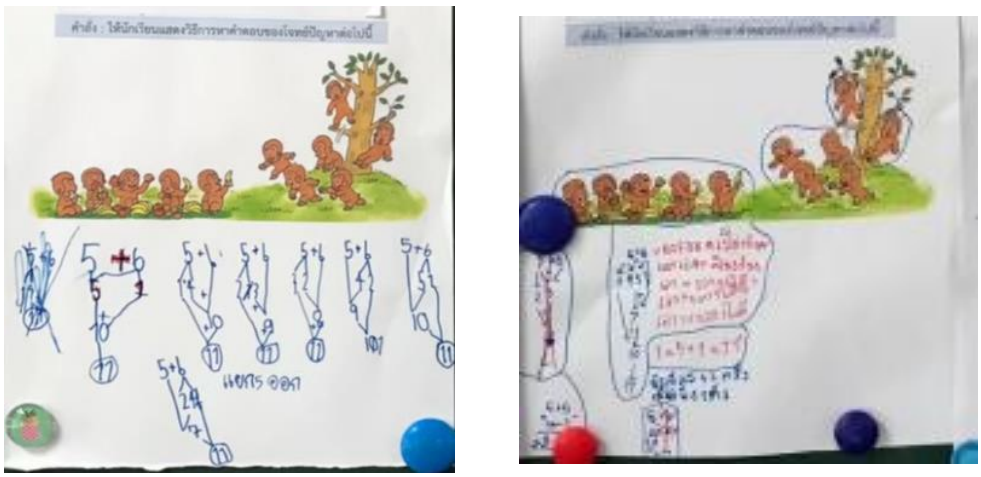
4) Separation Option

Separation option is the separating the number. Students will first determine which numbers are easier to separate. From participant observation, it indicated that students always choose to separate the smaller numbers. And, they usually show the separation of the number of added numbers. The picture 4 is situation of 5+6. Students separated 6 as 5 and 1. They did 5 plus 5 becoming 10. Then, they did 10 plus 1 becoming 11.

Picture 4: Students' Separation Option

5) Separation plus addition

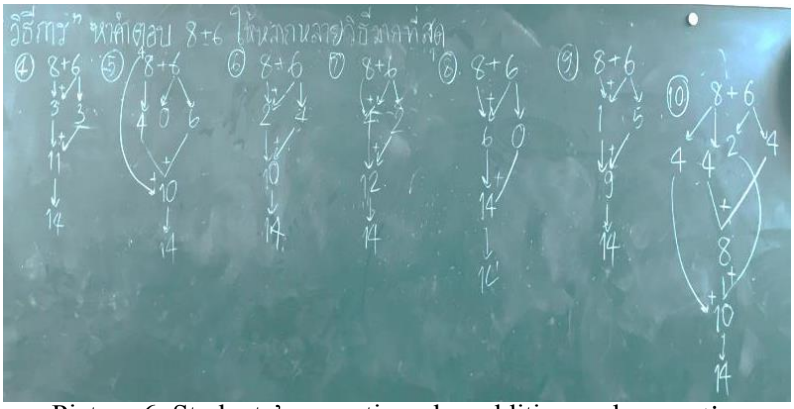
For separation plus addition, students find the number to separate first as well as the positive separator. When students find that separating positive ones is easier, they will separate the number of positive numbers. And, a number that is separated from the positive addition to add up the positive number in full ten. For example, the picture 5 is the situation of 5+6. Students separated 5 as 1 and 4. Then, they did 4 plus 6 because they had sense of making ten. Then, they did 1 plus 10 becoming 11.



Picture 5: Students' Separation plus addition

6) Separation plus addition and separation.

Separation plus addition and separation could be seen as the picture 6. It found that situation of 8+6 could be solved as various kinds of separation plus addition and separation. For example, students separated 8 as 4 and 4 and separated 6 as 2 and 4. Then, they did 4 plus 4 becoming 8. And, they did the making ten by 8 plus 2. Then, they did 10 plus 4 becoming 14.



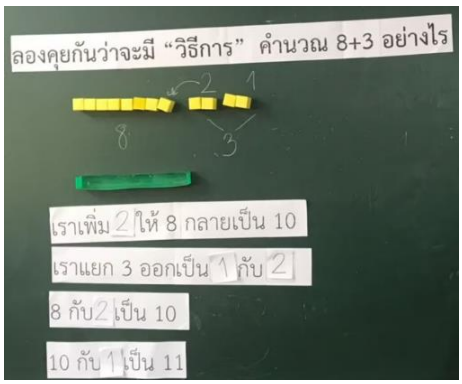
Picture 6: Students' separation plus addition and separation

3.2 Some basic ideas for enhancing students' developing strategies of solving problem on addition

The reflection of lesson study suggested some fundamental strategies of solving problem on addition. Regarding on students' performance of solving problem on addition, the members of lesson study concluded some basic ideas for enhancing students' developing strategies of solving problem on addition. These included 1) practicing about making ten full by using blocks and rail blocks, 2) practicing about using blocks and symbols, and 3) teachers' providing of encourage questions to find other ways of addition.

1) Practicing about making ten full by using blocks and rail blocks

Using blocks and rail blocks could enhance students to develop their strategies of solving problem on additions. Blocks and rail blocks may help students to make senses the strategies of counting, total counting, making ten separation option, separation plus addition, and so on. The picture 7 showed that how the blocks and rail blocks support students to make sense of 8+3 situation. The 8+3 situation asked students that "Let we talk about how many strategies to solve problem of 8+3". Then, two groups of blocks were provided – one was 8 blocks and another was 3 blocks. Separate the group of 3 blocks as 2 blocks and 1 block. Then, gave 2 blocks to 8 blocks becoming10 – making ten. Then, they did 10 + 1 = 11.

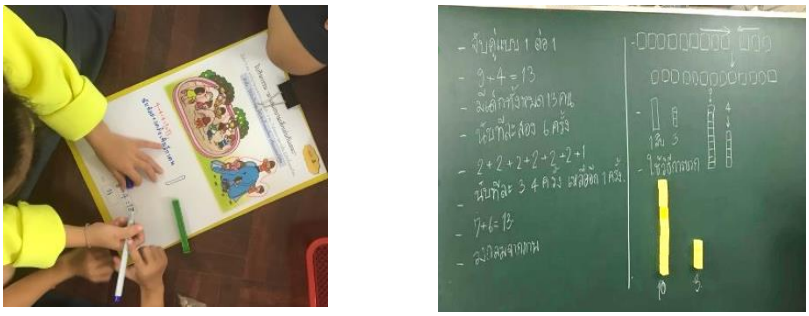


Picture 7: Using blocks and rail blocks to make senses the strategies of counting

2) Practicing about using blocks and symbols

Using blocks help students to make sense the representation of addition by finding total number. And, it could connect to symbols about addition. The participant observation allowed us to learn that the concept was developed to use symbols such as positive separator, positive separator and positive separator and positive separator. Then,

in later activities, students will not use blogs and counting. But, students used positive identification methods separating positive and separating positive and separating. Then, they used the blogs in order to explain teachers more ideas about the group of 10 and another when the teacher kept giving students opportunity for further thinking. The picture 8 showed that students provided the blocks and symbols to represent and connect concept of addition.



Picture 8: Using blocks and symbols to represent and connect concept of addition

3) Teachers' providing of encourage questions to find other ways of addition

In order to enhance students to construct their own strategies of solving open problem on addition, teachers needed provide some encourage questions. These questions may help students monitor and regular their thinking for solving problem. The examples of some encourage questions could be provided as following:

"Try to think in a variety of ways"
"Is there any other way to think?"
Is something that encourages students to find ways to add and find the total number of members?
Are there any strategies that differ from others?
Are there any strategies that are not same as others?
Do you want to add up more ideas?
Besides your presentation, any other strategies could do?
Please share your ideas that close to your friend presentation.

4. Conclusion

The open approach learning activities of the unit 2 of addition enhanced students to learn various ways of solving open ended problems about addition. Through students' presenting of various ways of solving open ended problems about addition, it indicated that students would make sense the process of mathematics about addition. The open approach allowed teachers to provide various media in order to construct mathematical representation and connection in concept of addition. Media for open approach is important for students' problem solving and constructing meaning of mathematical representation and connection. The lesson study supported teachers to provide the appropriate encourage questions for students' developing strategies of solving opened problems.

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