Asia Research Network Journal of Education
Vol. 1, No. 3, pp. 121- 135, Sep. – Dec. 2021

Development of Problem-Solving Approach Lesson Plans in Geometry

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Received: 10 Aug 2021 Revised: 07 Nov 2021 Accepted: 14 Nov 2021

Abstract. This article is the first of the two-part study of development and implementation of a problem-solving approach lesson plans in Geometry. In this article, the development and evaluation of two lessons utilizing problem approach each on Ratio and Proportion and the Proportional Segments and Basic Proportionality Theorem for Grade 9 is described. Starting from the mapping of competencies, the development of problems, anticipation student solutions and class discussions are detailed. The developed lessons are evaluated by four mathematics experts who are trained in teaching mathematics through problem solving and fourteen pre-service mathematics teachers using a scoring rubric. The scoring rubric emphasized both the technical aspects of the lesson plan and the instructional procedure that followed the problem-solving approach patterned from the teaching mathematics through problem-approach (TMPS). Several revisions were noted to improve the utility of the approach such as that of presenting the problem, multiple anticipated solutions, development of concept through discourse, and generalization. The evaluated lessons are noted to be ready for classroom implementation.
Keywords: lesson planning, problem-solving, problem-solving approach, development and evaluation

1. Introduction

Mathematics is ever around us, in technology, economy and in every aspect of human society. However, what is difficult for teaching mathematics today is that the presence of mathematics around us is not evident, thus hard to appreciate. By improving Mathematics instruction, the development of mathematical skills may also be improved to make the students more capable problem solvers. Among the participating countries during the 2003 TIMSS assessments, Philippines ranked fifth from the bottom in eighth-grade Mathematics achievement. And on a report by UNESCO in Challenges in Mathematics Education it stated that Filipino students perform relatively lower compared to its neighboring countries in mathematics achievement. Furthermore, results in the National Achievement Test for Mathematics reflect poor mastery in the field of Mathematics for high school level. These data and observations reflect an ever-present problem but, these are reported before the implementation of the K-12 curriculum. Now that Philippines adopted this curriculum, improvement in mathematics teaching and learning is ought to be expected. Problem solving should have a prominent role in the mathematics education of the K-12 students (Cai & Lester, 2010; DOST-SEI & MATHTED, 2011) for mathematics learning is dependent on the problem solving and problem solving is dependent on the mathematics (Selmer and Kale, 2013). This raises the question: how should teachers teach mathematics to develop problem solving skills among the students? And how should students learn mathematics?

Many approaches have been employed in teaching mathematics in the history of mathematics education. And recently, teaching through problem solving gained the attention and strong support among researchers, educators, and teachers; and there is a widespread agreement that this approach holds a promise in fostering mathematics learning (Schroeder & Lester, 1989 as cited by Cai, 2003; Ulep, 2010; Takahashi, 2021). But, as a teaching approach, it is relatively new that it has not been a subject of much research (Cai, 2003). Specifically in the Philippine context, while there are initiatives towards the adoption and use of teaching through problem solving, it is not yet widely used (Buan, Medina & Liwanag, 2021). Thus, the researchers, wishes to study this approach in teaching mathematics by answering the research question: How are the problem-solving approach lessons developed?

2. Review of Literature

The literature review highlights the importance of problem solving in the instruction of mathematics and how mathematics can be taught using problem-solving approach effectively. The last section provides the theoretical framework defining the approach from different scholars and the general method of teaching mathematics through problem solving.

Foundation of Problem-Solving Approach

According to an article published by NCTM (2010), problem solving plays an important role in mathematics. In another article by Selmer & Kale (2013), mathematics is dependent on the problem solving and the problem solving is reliant on the mathematics. Yet the question of how to incorporate problem solving into mathematics curriculum may still be a question to many mathematics teachers (Cai & Lester, 2010). Similarly, Lester (2013) addressed the issue of “whether problem solving is intended to be an end result of instruction or the means through which mathematical concepts, processes, and procedures are learned” (p.246) which he further argued that both has merit, that is, as an end and as a means. Takahashi (2021) also asserts that there is a need to move from the lecture method to promote the learning of mathematical thinking and problem solving.

Even though teaching through problem solving is relatively new in the history of problem solving in the mathematics curriculum (Cai, 2003), the literature and studies on problem solving and learning mathematical problem solving may provide useful suggestions to educators and policy makers (Cai & Lester, 2010). In fact, standards being established suggest that problem solving should be the “primary” means of achieving mathematical understanding (Matheson, 2012; & Takahashi, 2021).

Cai and Lester (2010) on NCTM’s research brief noted that teaching through problem solving is theoretically founded. Contradictory of the traditional method of teaching, which the teacher is the dispenser of knowledge and the students passively “learning”, this approach is founded in the constructivist and sociocultural perspective of learning which focuses on the learner's’ thinking about learning and their “construction” of meaning through their experiences. Constructivism is one of the learning theories that posits that “human learning is constructed, that learners build new knowledge upon the foundation of previous learning” in contrast the traditional learning which “… is the passive transmission of information from one individual to another" (Bada & Olusegun, 2015; p. 67). On the other hand, Lev Vygotsky in his Mind in Society, notes that social interaction is essential to learning and that learning implies development (Vygotsky, 1987). It is in this sociocultural perspective of learning that collaborative learning is seen as an effective means of facilitating learning even in the context of developing problem-solving skills (Medina, Buan, Mendoza & Liwanag, 2019). In the Problem-Solving approach, follows its principles of constructivism in the sense that the learners are the ones "building" their knowledge, the active agents in the learning process. As learners solve problems in small groups, and facilitated discussion in the whole class, it also follows the principles of sociocultural learning theory.

Effectiveness of Problem-Solving Approach

The study of Ali et. al. (2010), utilizing pretest-posttest design found out that students taught with "problem solving method" achieved better than those taught with the "traditional method" of teaching mathematics. Also, Samuel Onyinyechi Nneji (2013) investigated the effects of Polya George's Problem-Solving Model (POGPROSMO) on the achievement and retention of the students in Algebra, compared to the students taught of the same topics using expository method, i.e., the traditional discussion method. He concluded that students taught using POGPROSMO achieved and retained better in Algebra. Furthermore, Perveen (2010) of Pakistan conducted a similar study entitled "Effect Of The Problem-Solving Approach On Academic Achievement Of Students In Mathematics At The Secondary Level", where he also concluded after the treatment, the experimental group of students taught with "problem-solving approach" significantly achieved better. All these findings support NCTM's (2003) reasons why teachers should teach through problem solving, to wit: (1) It helps students understand that mathematics is a making-sense process, and (2) It deepens student's understanding of underlying mathematical ideas and methods (Matheson, 2012). However, the question of whether the students actively engage in the process of teaching through problem solving remains.

Structure of Problem-Solving Approach

By the preceding literatures, it is worth to note that teaching "through" problem solving is indeed beneficial supporting Cai's (2003) statement in her article "What research tells us about teaching mathematics through problem solving," that this approach is receiving support from researchers, teachers, and educators despite the lack of further research. By tackling the issues relating to teaching mathematics through problem solving, NCTM came up with an image of how teaching mathematics through problem solving should look like. In doing so, certain points are highlighted: (1) developing or choosing worthwhile problem, (2) the role of the teacher and (3) organizing a discourse (Cai & Lester, 2010).

Several other authors laid similar pictures of what teaching through problem solving is about. Fi and Degner (2012) described "Teaching through Problem Solving" (TtPS) as "pedagogy that engages students in problem-solving as a tool to facilitate students' learning of important mathematics subject matter and mathematical practices" (p. 455) laying some key "moves" to describe the approach. (1) Posing a worthwhile problem with its mathematical complexity, (2) letting the students explore the problem, build conjectures and letting them share their work, (3) from their shared work, focus on the big mathematical idea or concept, (4) build the concept or idea from the contribution of the individual or groups, pointing out mistakes and misconceptions in the process and (5) closure: provide time for students to reflect on what they learned (Fi & Degner, 2012). Several other research paint a similar picture (e.g. Ulep, 2010; Lester, 2013; Matheson, 2012; Donaldson, 2011; Selmer & Kale, 2013), though Selmer and Kale (2013) used "Teaching Mathematics through Problem Solving" (TMPS) to describe the approach. Takahashi (2021) pointed that the *neriage* or the comparing and discussion is the heart of the TTP. This part of the lesson corresponds roughly to the "building of concept" or the "processing of the lesson/discourse". The figure below is the framework adopted by DOST-SEI Project Science Teachers Academy for the Regions training as mentioned in the work of Buan, Medina, and Liwanag (2021).

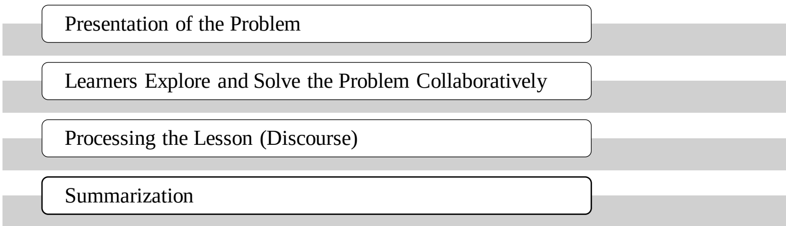


Figure 1. Problem Solving Approach, also known as the TtPS or TMPS

One of the recommendations of the literature cited (Cai, 2003; Donaldson, 2011) suggested further research on the topic problem solving, especially as a means of teaching. And Lester (2013) pointed that teacher planning has been given too little attention as a factor of importance in problem-solving instruction research.

In light of the aforementioned literatures, the researchers were led to focus on the development and validation of lesson plans, seeing it as a means of addressing the above-mentioned issues relating to teaching problem solving and more importantly to teacher planning. Moreover, the researchers, desiring to know and contribute to the present literatures, wishes to gather data on student beliefs, perception, and performance after the implementation of the validated lesson plans. And lastly on whether there shall be an improvement in the problem-solving skills of the students.

3. Methodology

This study employs mixed methods design, where both qualitative and quantitative data are collected in the development process. Qualitative data are gathered through the class observations and reflections of the researchers, knowledgeable others, and evaluators, while quantitative data are gathered using the evaluation rubric for the lesson plans.

Development of Problem-Solving Approach Lesson

The evaluation of the lesson plans utilizing problem-solving approach, is undertaken by a group of evaluators composed of four (4) mathematics experts and fourteen (14) pre-service teachers. One (1) mathematical expert, one (1) pre-service teacher and one (1) in-service teacher from the school where the implementation took place formed the Lesson Study group and observed the implementation of the lesson plans during the final implementation. The try-out was conducted with one (1) section of Bachelor of Elementary Education – English (BEED-English). The BEED-English students are purposively selected with the purpose that they have exposure to the problem-solving approach at the university level. They serve as the pilot students for the developed lesson plans before actual field implementation.

In making of the two (2) lessons plans, the process is diagrammed in Figure 2. The first step is looking into the K-12 Mathematics curriculum guide and identified the topic teachable through problem solving approach for the implementation period. Following is the identification of the topic is the identification of K-12 standards and learning competencies with the formulation of lesson plans objectives.

The making of the Table of Specification followed. From there, the researchers drafted the lesson plans, and were evaluated by the mathematics experts and pre-service teachers, after. A cycle of revisions was undertaken following the evaluation of the lesson plans to produce the lesson plans ready for try outs. An initial try-out was undertaken with pre-service teachers as the subject of implementation. Another cycle of revisions followed to produce the lesson plans ready for the implementation.

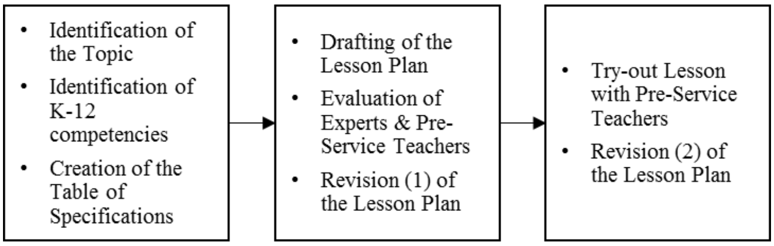


Figure 2: Development Phase

Data was collected primarily from observation of the lesson study group and the post lesson discussion that follows the pilot implementation. The data collected came from the reflection of the implementer/researcher, from observer/researcher and a knowledgeable other, who is the mathematics teacher of the pilot class (BEED students).

4. Results and Discussion

The following section discusses the results of the study, namely the development process, and the final lesson problem situations. In the development process, it details the relevant information of identifying the topics and standards, the drafting of the lesson plan based on the framework of teaching through problem solving. After the development is the evaluation of the drafted lesson plan where mathematics experts, who have substantial experience in the said approach, evaluated the lesson plans. Preservice teachers, who are also exposed to the approach in the university evaluated the drafted lessons as well. Revisions are highlighted based on the comments and the observations of the lesson study group, composed of the researchers and a knowledgeable other during the pilot lesson implementation. The validated lesson plans' problem situation are also discussed.

4.1 Development of the Problem-Solving Approach Lesson

Lesson Plans utilizing problem approach is a semi-detailed lesson plans for teaching Ratio and Proportion and the Basic Proportionality Theorem in Grade 9. A lesson plan utilizing problem solving approach does not follow the conventional structure, but rather it utilizes a problem as a springboard to the student activity and the meaningful discourse that follows. Furthermore, it reflects some possible solutions that are instrumental in developing the concepts.

Identifying the topics

The topic "Ratio and Proportion" and "The Basic Proportionality Theorem" are topics preceding the topic "Triangle Similarity". Triangle Similarity is one of the important concepts in Geometry. The selection of the topic for this study does not necessarily follow that the topic is a least learned or least mastered since the main objective of this study is the approach to which the topic is delivered – the Problem-Solving Approach of teaching Mathematics.

Identifying K-12 standards and Formulation of Lesson Plan Objectives
The K-12 Mathematics Curriculum provides a sound basis for Mathematics instruction. It also provides a list of sequenced concepts and skills necessary for the learners to cope up with the demands of life. The K-12 Curriculum Guide for Mathematics also sets the standards for teaching Mathematics in all grade levels.

To ensure that the objectives formulated for the Lesson Plans agree to the National Standards set through the K-12 Mathematics Curriculum Guide, the standards were identified and mapped as shown in the following table:

Table 1: Identifying & Mapping of Standards from the K-12 Curriculum

K-12 Math Learning Competencies	**Learning Objectives for the Lesson Plans**
• Describes a Proportion	• Describe a proportion
	• Discuss some properties of proportions
• Solves problems that involve triangle similarity and right triangles.	• Solve problems involving proportions in real-life situations
	• Use ratio and proportion in solving word problems involving proportional segments.
• applies the fundamental theorems of proportionality to solve problems involving proportions.	• Discuss the concept of the Basic Proportionality Theorem
	• Apply the concept of the Basic Proportionality Theorem

Drafting the Lesson Plan
A lesson plan utilizing Problem Solving Approach of teaching does not follow the conventional format of a detailed lesson plan. It has its own set of parts not compromising the essentials of the conventional one. Specifically, some of the major parts of the lesson plan are as follows:

1. “Technical Details” – This section contains the following details: (1) Lesson title, Grade Level and Strand; (2) Prerequisite Concepts and Skills; (3) Learning Competencies addressed or the Objectives; and (4) Instructional Media that is, the References and Instructional Materials.
2. About the Lesson – This section of the lesson plan gives the overview of the whole instructional plan, its purpose and relevance. It encapsulates the whole procedure and as well as the desired outcomes as reflected in the Learning competencies or the Objectives.
3. Instructional Procedure – This section contains the following subsections:
    a. The problem – The problem is selected with the help of the Checklist for a Worthwhile Problem, adapted from Lappan and Phillips (1998) as recommended by Cai and Lester (2010) in an NCTM article. The problem serves as the springboard of the discussion. Hence, it must contain all essential qualities of a worthwhile problem.
    b. Possible Answers and Solutions – Varied solutions may come from the students which can lead to the concept, or skill addressed is reflected in this section. The lesson plan utilizing Problem Solving Approach must contain several varied solutions and shall be used as the basis for the Development of Concept.

c. Development of Concepts – Developing the concept involves a discourse. A teacher-student-student interaction, through questioning, creatively designed to build and develop the concept. It must effectively relate the anticipated solutions reflected in the plan, encourage critical and higher order thinking and give learners opportunities to ask questions.
d. Closure/Summary – Also known as Closure and Generalization, the learners must generalize, through questioning, the concept and skills addressed in the plan. It includes the revisiting of the purpose of the lesson and emphasis on the what the learners have learned.
e. Assessment – In the lesson plan, the formative and summative assessments are defined, showing clear relationship to all objectives addressed in the lesson. It is a means of knowing whether the objectives are achieved.

Evaluation of the Problem-solving Approach Lesson Plans

The rubric for the lesson plan evaluation contains 8 components, with the Problem component weighted 40% and the average of the rest of the components 60%. The 8 components are subdivided into *About the Lesson (Objectives, Materials, Introduction)*, and the *Instructional Procedure (Problem, Possible Solutions, Development of the Concept, Assessment, & Closure & Generalization)*. This "Rubric for Lesson Plan" (2012) is adopted from Alvernia University, revised to cater to the proposed problem-solving approach instructional procedure.

Table 2: Summary of Ratings from Math Experts & Pre-Service Teachers

Category	Descriptor	Lesson 1		Lesson 2	
		ME (n=4)	PT (n=14)	ME (n=4)	PT (n=14)
Objectives	Lesson objectives are clear & measurable, specifically aligned to the K-12 standards; learning progression is evidenced	**3.8** *Very Satisfactory*	**3.8** *Very Satisfactory*	**3.8** *Very Satisfactory*	**3.8** *Very Satisfactory*
Materials	Detailed list of Materials is provided for both teacher and students. All handouts, both teachers created and those from other resources, are referenced in the procedures and attached to the lesson plan	**3.5** *Very Satisfactory*	**4.0** *Very Satisfactory*	**3.5** *Very Satisfactory*	**3.9** *Very Satisfactory*
Introduction	Introduces the lesson by sharing purpose, relevance; with clear overview of the student activity	**3.8** *Very Satisfactory*	**4.0** *Very Satisfactory*	**3.8** *Very Satisfactory*	**3.9** *Very Satisfactory*
Problem	Exceeds all 4 essential qualities and includes many of the qualities reflected in the checklist. Lappan and Phillips (1998)	**3.3** *Very Satisfactory*	**4.0** *Very Satisfactory*	**3.3** *Very Satisfactory*	**4.0** *Very Satisfactory*
Possible Solutions	Reflects more than 3 solutions/means of solving the problem	**4.0** *Very Satisfactory*	**4.0** *Very Satisfactory*	**4.0** *Very Satisfactory*	**3.8** *Very Satisfactory*

Table 2 (Cont')

Category	Descriptor	Lesson 1		Lesson 2	
		ME (n=4)	PT (n=14)	ME (n=4)	PT (n=14)
Development of Concept	The development of concept effectively relates the anticipated solutions The students are given more than adequate opportunity to ask questions and are encouraged to interact with their classmates The teacher often asks higher order thinking questions	**3.3** *Very Satisfactory*	**4.0** *Very Satisfactory*	**3.3** *Very Satisfactory*	**3.9** *Very Satisfactory*
Assessment	Formative and summative assessments are defined, showing clear relationship to all objectives address in the lesson	**3.5** *Very Satisfactory*	**3.8** *Very Satisfactory*	**3.5** *Very Satisfactory*	**3.9** *Very Satisfactory*
Closure and Generalization	Students review the lesson by summarizing and/or sharing what they learned, teacher revisits the purpose for the lesson	**3.0** *Satisfactory*	**3.8** *Very Satisfactory*	**3.0** *Satisfactory*	**3.7** *Very Satisfactory*
Weighted Means		**3.4** *Very Satisfactory*	**3.9** *Very Satisfactory*	**3.4** *Very Satisfactory*	**3.9** *Very Satisfactory*

Legends: ME = Mathematics Experts, PT = Practice Teachers

Table 2 summarized the mean ratings of the four (4) mathematics experts and fourteen (14) mathematics pre-service teachers on the developed problem-solving approach lesson plans. Most of the mean ratings of the components are rated as very satisfactory, save for the closure and generalization where among the ME's only garnered an average rating of satisfactory. Notice that the ratings of the Mathematics Experts are considerably lower in the problem-solving approach instructional procedure, components 4-8, and the researchers saw that there is a great need to revise these parts to validate the approach used. The following sections discuss the changes and revisions of the key PSA components.

Other revision suggestions are also taken into consideration such as the length of the lesson and the grammar and mechanics, as the following evaluators noted: "*Please implement the editing corrections" – ME3, "There are typos and grammatical errors present. Consider revising." – PT10,* and "*I think one (1) hour is not enough for this lesson plan, consider shortening." – PT1,* the clarity of objectives and the inclusion of instructional materials as some evaluators noted: "*Teacher's role in implementing a lesson utilizing problem solving approach is a facilitator, hence, giving direct definitions are "not necessary" – ME3, "Use manipulatives." – ME1,* and *"Consider the resources available, using a projection system consumes time." – ME4.*

Changes and Revisions on the PSA Instructional Procedure

There are several revisions for the problem-solving approach lesson plans that was developed. Table summarizes these revisions based on the evaluator's expert feedback and comments. For both lessons, there are two major revisions.

Table 3: Summary of Lesson Revisions for Lesson 1: Ratio & Proportion

Lesson Component	Before	Comment	Action
Revision 1			
Problem	The problem for Lesson Plan 1 is about baking; a very simple problem of ratio and proportion.	No longer a Problem for a grade 9. – ME1 A good problem for grade 7, but not for grade 9. – ME4	The problem is replaced with a real-life scenario problem appropriate for a grade 9 level.
Development of Concept	Development of concept does not include the student presentation of the anticipated solutions.	Include in the development of concept how the students would present their solutions. – ME1	The explanations of the students for the anticipated solutions are included in the development of concept.
Assessment	Assessment is limited to knowledge and comprehension levels only. No real-life scenario problem.	Include a real-life scenario problem in your assessment. – PT6 Maybe, one (1) problem is enough for the assessment. – PT8	Assessment items were also reduced to one or two items with a real-life scenario problem.
Revision 2			
Development of Concept	The notations are denoted by letters a, b, c and d.	Revise notations on setting up ratios and proportions. Do not include in the Lesson Plan the "recall", that is, do not readily assume that pre-requisite skills are attained.	Notations are changed such that "Andrew's Height" is given as "Ha" read as "Height of Andrew" and so on.

Above, the table 3 summarized the changes and revisions the evaluators suggested to improve the drafted lesson plan 1. There are major suggestions that related to the Problem-Solving Approach instructional process, i.e., on the problem, the development of concept and the assessment. The Problem is very crucial to the engagement of the students in the problem-solving process and looking at the criterion laid out by Lappan and Phillips (1998) as cited by Cai & Lester (2010), the problem needs improvement, as commented by the Math Expert evaluators, *"No longer a Problem for a grade 9." – ME1* & "*A good problem for grade 7, but not for grade 9." – ME4.* From the problem that focused on simple proportion of ingredients, the problem was improved to now involve using proportion in computing for shadows (See Appended Lesson Plan 1).

The second component in the instructional procedure that was revised is the *development of the concept*, which corresponds to the whole class discussion and processing of the lesson (discourse). The math expert evaluator noted, *"Include in the development of concept how the students would present their solutions." – ME1* This evaluation is important as the anticipated presentation of the students of their solutions and ideas will be the basis for the building of the concept and the construction of the knowledge as a class.

Lastly, on the assessment it was suggested by the pre-service teacher-evaluators to "*Include a real-life scenario problem in your assessment." – PT6* & "*Maybe, one (1) problem is enough for the assessment." – PT8.* This evaluation is consonant to the practice of making the lesson more relevant by providing real-life context, and thus was incorporated.

On the second revision, after the try-outs to pre-service teachers, the researchers noted that there was a need to improve the notations during the development of the concept and add concept "recall" deliberately to check on the students understanding or past learning as noted: *Revise notations on setting up ratios and proportions. Do not include in the Lesson Plan the "recall", that is, do not readily assume that pre-requisite skills are attained.*

Table 4: Sample Changes Implemented in Lesson 1

Drafted Lesson Component	**Revised Lesson Component**
Problem	
A. Problem Present the problem. *Instructions: Solve the word problem in many ways as you can. Show all solutions in the answer sheet provided. Work with your group.* **You are tasked to mix some dough for your baking class. Your teacher instructed you to mix the ingredients in the following manner. For every 2 cups of flour, add 1 egg, 3 spoon-full of butter and 150 ml of water.** Ask the following questions: → In the situation class, what is the task given to you? *Mix some dough.* → What are the ingredients that you should mix? *Flour, eggs, butter and water.* **Then, if you poured in 6 cups of flour in the mixing bowl, (1) how many eggs should be added? (2) How much butter? (3) How many ml of water?** The students will be given a maximum of ten (10) minutes to solve the problem.	**A. Present the following problem:** **Andrew is 5 feet tall and casts a shadow, 8 feet long. At the same time of the day, a tree casts a shadow of 32 feet long. What is the height of the tree in feet?** **Anya says the height of the tree is 20 ft while Jan says that the tree measures 30 ft. Who is right Anya or Jan?** Ask the following questions: What is asked in the problem? We are to find the height of the tree and to find out who is correct, Anya or Jan. Ask the students to solve the problem in as many ways as they can within the time limit of 10 minutes.
Development of Concept	
C. Development of Concepts The students must pass their worksheets in front and let one member each group present their best solution to the class by showing and briefly explaining it briefly. → Focus to question "(1) How many eggs should be added?" → How many eggs should be added? Three (3) eggs must be added to the flour to meet the proper mixture instruction. Answers reflected on the board may reflect one or more that of those discussed as possible solutions. Check for solutions similar to the concept of ratio or that of fraction and build the lesson from there. → Focus now on a solution which utilizes **Drawing, Tallying, Tabulating or Making a Diagram.** Compare and contrast each solution from the other.	**C. Development of Concepts** Let the students explain their answer. For Solution 1 *Andrew's height is 5 feet, and he casts a shadow 8 feet long. At the same time, a tree of unknown height casts a shadow 32 feet long. Observe that we can "fit" 4 of Andrew's shadow to be of the same length of the shadow casted by the tree. Hence, the height of the tree must also be 4 times that of Andrew's height. Therefore, the tree's height is 20 feet. This makes Anya's claim correct.* For Solution 2: *Drawing a scaled illustration would mean setting a scale beforehand. Setting the scale 2 feet = 1 cm is arguably a practical scale due also to the limitation of the standard ruler's length. Now, we reflect the scaled measurements; that is, 5 feet = 2.5 cm, 8 feet = 4 cm, 32 feet = 16 cm. Following a similar logic from Solution 1 would lead us to the fact that 16 cm is just 4 parts of 4 cm each. Hence, the height of the tree must also be 4 parts of Andrew's height. Meaning, in the illustration, the tree must have a height of 10 cm. Converting it back to feet, we have to recall the scale; 2 feet = 1 cm and x = 10 cm. Now, we multiply 10 to the first equality, that is 10(2 feet = 1 cm) so that we can achieve an equality with 10 cm. Finally, 20 feet = 10 cm. Therefore, x = 20 feet.* … Ask: So how high is the tree? The tree is 20 feet high.

Table 4 exhibits the changes suggested by the expert evaluators on the appropriateness of the problem. From the simple context, the complexity of the situation is increased such that the students are required to build on knowledge of a more advanced modelling of the situation. On the development of the concept, it is important to be able to anticipate how might the students explain or articulate their ideas, and from there, the teacher facilitates the discussion of building the concept. Comparing from the drafted part to the revised version, the teacher can be more prepared in the discussion knowing the anticipated response of the student on their solutions.

Table 5: Summary of Lesson Revisions for Lesson 2: Proportional Segments & Basic Proportionality Theorem

Lesson Component	Before	Comment	Action
Revision 1			
Possible Solution	Some possible solutions are very similar and uses the same concept.	What is the difference between these two solutions? It is redundant. – ME2	The possible solutions mentioned are fused and another solution was added.
Development of Concept	Development of concept does not include the student presentation of the anticipated solutions.	Include in the development of concept how the students would present their solutions. – ME1	The explanations of the students for the anticipated solutions are included in the development of concept.
Assessment	Assessment does not include a real-life problem.	Include a real-life scenario problem in your assessment. – PT6 Maybe, one (1) problem is enough for the assessment. – PT8	Assessment items were also reduced to one or two items with a real-life scenario problem.
Revision 2			
Problem	The problem involved the standard measure of feet and at the same time involved the term “base of the ladder” which made the second question quite confusing.	Revise the problem such that it won’t be confusing.	The standard measure was changed to meter such that the “base of the ladder” can be simply described as “foot of the ladder”.

Table 5 summarizes the revisions for the lesson plan 2. The problem utilized is still rated *very satisfactory* but there are no major revisions proposed by the evaluators both the mathematics education experts (MEs) and the pre-service math teachers (PTs). However, one of the MEs, noted that the anticipated solutions for the problem are *“... redundant.” -ME2*, that is, only the mode of representation different but the same concept is applied. Secondly, like the lesson plan 1, it is also requested that the presentation of solutions for the solutions by the students will be included in the plan. Lastly, on the 1st round of evaluation, the assessment items need to be reduced in items, with a real-life context.

After trying out the lesson to some pre-service teachers, it was noted that students were quite impeded in solving the problem due to the term “base”, as noted by one observer, “*The problem involved the standard measure of feet and at the same time involved the term “base of the ladder” which made the second question quite confusing.”*

In the second lesson, focusing on the conceptual development of proportionality theorem, the ME2 suggested to revise the possible solutions to clarify the concept utilized. Furthermore, like lesson 1, student explanations for their solutions are also anticipated so that the teacher can better prepare for the concept development. See Appendix 1 for the link of the Lesson Plans after the 2nd revision.

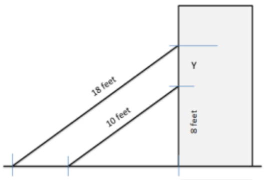
Table 6: Sample Changes Implemented in Lesson 2

<table>
<tr><th>Drafted Lesson Component</th><th>Revised Lesson Component</th></tr>
<tr><td colspan="2">Possible Solutions</td></tr>
<tr><td>
(1,2) Ratio and Proportion. Utilizing ratio and proportion is one of the easiest ways in solving this problem. By simply making an analogy of the two ratios and by using the fundamental law of proportion, then the missing term can be easily determined.

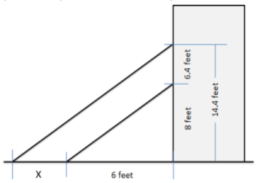
Length of Ladder : Reach on the wall. Given that they are make equal angles with the horizontal, then we can identify ratio $10\ ft : 8\ ft = 18\ ft : (8 + x)ft$. By the fundamental law of proportionality, $80 + 10x = 144 \rightarrow 10x = 64 \rightarrow x = 6.4$

Reach on the wall : Distance of base. Given that they are make equal angles with the horizontal, then we can identify ratio $14.4\ ft : (6 + y)ft = 8\ ft : 6\ ft$. By the fundamental law of proportionality, $8(6 + y) = 86.4 \rightarrow 8y + 48 = 86.4 \rightarrow 8y = 38.4 \rightarrow y = 4.8$

(1,2) Illustration with Ratio and Proportion. The solution may appear exactly the same as mentioned in the preceding method. The difference is that it is illustrated, both the situation and the variables.



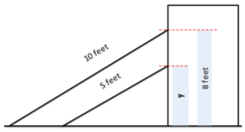
Length of Ladder :
Reach on the wall. Given that they are make equal angles with the horizontal, then we can identify ratio $10\ ft : 8\ ft = 18\ ft : (8 + y)ft$. By the fundamental law of proportionality, $80 + 10y = 144 \rightarrow 10y = 64 \rightarrow y = 6.4$



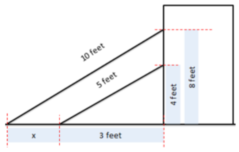
Reach on the wall : Distance of base.
Given that they are make equal angles with the horizontal, then we can identify ratio $14.4\ ft : (6 + x)ft = 8\ ft : 6\ ft$. By the fundamental law of proportionality, $8(6 + x) = 86.4 \rightarrow 8x + 48 = 86.4 \rightarrow 8x = 38.4 \rightarrow x = 4.8$
</td><td>
2. Ratio and Proportion:

Utilizing ratio and proportion is one of the easiest ways in solving this problem. By simply making an analogy of the two ratios and by using the concept of proportion, then the missing term can be easily determined.

Length of Ladder : Reach on the wall. Given that they are make equal angles with the horizontal, then we can identify ratio $5\ ft : y\ ft = 10\ ft : 8\ ft$. By using the concept of proportionality, $10y = 40 \rightarrow y = 4$.



Reach on the wall : Distance of base. Given that they are make equal angles with the horizontal, then we can identify ratio $8\ ft : (x + 3)\ ft = 4\ ft : 3\ ft$. By using the concept of proportionality, $4(x + 3) = 24 \rightarrow 4x + 12 = 24 \rightarrow 4x = 12 \rightarrow x = 3$



</td></tr>
<tr><td colspan="2">Development of Concept</td></tr>
<tr><td>
C. Development of Concepts
The students must pass their worksheets in front and let one member each group present their best solution to the class by showing and briefly explaining it briefly.
Focus to question (1)
→ Ask: How much further up the wall does the 18' ladder reach? 6.4 feet.
The answers reflected may have Graphical Solution or theoretical (including but not limited to Ratio and Proportion). Compare and contrast the two methods of solving.
→ Ask: What are the similarities and the differences of the solutions presented? Similarities include (not limited to) the use of illustration, the use of ratio and the result (or closely related). The differences include time consumption and use of theory or concept.
→ Stress, if stated, the limitation of the graphical solution. If not stated nor answered, ask: What are the limitation of the graphical solution? Lacking standard measuring devices, e.g. ruler, protractor. Graphical solutions should be sketched as accurate as possible to minimize error. And lastly, aside from time consuming, it is less accurate compared to the theoretical solution.
Focus now on the theoretical solutions and focus on the illustrated situation with the results from the solutions.
</td><td>
C. Development of Concepts
Let the students explain their answer.

For Solution 1
First, we draw a horizontal line representing the ground. Second, we draw a vertical line to represent the wall. After this, with the scale established (1 cm : 1 foot) we measure 8 cm from the ground up the wall and from that point, measure 10 cm in such a way that the endpoint of the 10-cm segment is in the ground. Now, since we know that the second ladder measures 5 feet, 5 cm in our scale, we shall measure a 5 cm line which must be angled from the horizontal as the 10-cm line. The endpoints of the 5-cm line must be on the ground and in the wall. We can then measure the unknowns and then convert it back to feet. Hence, the reach of the 5-foot ladder is 4 feet while the 10-foot ladder's base is 3 feet farther than the base of the 5-foot ladder.

For Solution 2
…
Ask: So what is the reach of the 5-foot ladder?
The 5-foot ladder reaches 4 feet up the wall.
Then how much farther is the 10-foot ladder's base from the 5-foot ladder base if the latter's base is 3 feet away from the wall? The 10-foot ladder's base is 3 feet farther from the wall compared to that of the 5-foot ladder.
… (Questioning continues)
</td></tr>
</table>

4.2 The Validated Lesson Plan Problem Situations

There are two lesson plans developed that constitute one learning unit in teaching the introduction to Basic Proportionality Theorem. The first lesson, on ratio and proportion used the problem situation on reasoning about shadows and heights that intended to make students connect the concept of ratio and proportion to real life scenarios:

> *Andrew is 5 feet tall and casts a shadow, 8 feet long. At the same time of the day, a tree casts a shadow of 32 feet long. What is the height of the tree in feet?*
>
> *Anya says the height of the tree is 20 ft while Jan says that the tree measures 30 ft. Who is right Anya or Jan?*

This problem provided the students the opportunity to grapple and use their previous knowledge to come up with solutions, utilizing different methods. The goal of setting up a proportion was achieved by the students with the help of the teacher's facilitation during the pilot implementation.

Continuing from the first lesson, the 2^{nd} lesson on proportional segments and basic proportionality theorem used the problem of leaning ladders as the context to establish the theorem:

> *Two ladders, 10 meters and 5 meters long, lean against a vertical wall so that they make the same angle with the ground. The 10 meters ladder reaches 8 meters up the wall. How much does the 5 meters ladder reach?*
>
> *Suppose that the base of the 5-m ladder is 3 meters from the wall, how far from the wall is the foot of the 10-m ladder?*

Building up from the previous lesson where the students made a proportionality equation, the learners made use of the concept of ratio and proportion and other strategies to reach the establishment of the basic proportionality theorem. The full lesson plan may be accessed in the appended link.

5. Conclusion

This article details the process of the development of the Lesson Plans Utilizing Problem solving Approach. These lesson plans have some unique features compared to the generic lesson plan structure, that is, it uses a problem as a springboard to student activity and to be followed by a meaningful discourse. The lesson plans cover the topics Ratio and Proportion, Proportional Segments and Basic Proportionality Theorem. The lesson plans are aimed to provide guidance for teachers utilizing Problem Solving Approach to make mathematics classes more student-centered, collaborative, and interesting and fun.

The lesson plans are developed and validated through the following steps: (1) the identification of the topics Ratio and Proportion, Proportional segments and the Basic Proportionality Theorem are chosen as introductory lessons of Triangle Similarity; (2) the identification an mapping of K-12 standards and formulation of objectives; (3) drafting the lesson plan guided by the formulated Table of Specifications; (4) the lesson plans are validated through the evaluation of the mathematics experts and the pre-service teachers; (5) the lesson plans undergone revisions based on the evaluation; and (6) the lesson plans were tried out with pre-service teachers. The mathematics expert and the pre-service teachers evaluated the lesson plans all as Very Satisfactory. This implies that the lesson plans are recommended as a tool in teaching through problem solving.

The key finding of this discussion is the importance of the *problem presented* and the *development of the concept through discourse* or the *neriage or compare and contrast*. This is consonant to the works of Ulep (201), Fi & Degner (2012), Lester (2013) Matheson (2012), Donaldson (2011), Selmer & Kale (2013), Takahashi (2021), and Buan, Medina, and Liwanag (2021). Further investigation may be conducted with a more thorough observation of learners in the classroom implementation.

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Appendix 1

Lesson Plans access here:
https://drive.google.com/file/d/1P67Pjo1xIzEtf-QqXH7hLs9ruOQqQbu-/view?usp=sharing