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# Factor Analytic Method in Developing Scoring Rubric for Word Problems

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**Abstract.** The study aimed to develop a scoring rubric in solving word problems as an assessment tool that could be used to determine the student's level of performance, particularly in solving word problems. Scoring guides independently developed by teachers were consolidated using a priori criteria, then implemented to initial participants of prospective secondary teachers and first screened by PCA. Finally, the instrument was validated using the in-service teachers of the Eastern Zone with varied teaching experiences. Out of twelve criteria initially set, only two criteria emerged after subjecting to Exploratory Factor Analysis, which is the basis for a scoring rubric. After using the revised rubric, it was found out that the student's performance was on average level. Further findings and conclusions were discussed in this paper.

**Keywords:** Assessment tool; Scoring Rubric; Exploratory Factor Analysis; Problem Solving; Student's Performance

## 1. Introduction

Student's performance in solving word problems should be assessed, but mostly in the classroom situation, they focus on the correctness of the answer or the partial answer rather than the quality of the process on how to arrive at the answer. A more detailed measure is needed to properly evaluate the students' performance (Docktor et al., 2016). The problem-solving rubric would be of great help to facilitate students' development regarding their skills in solving word problems. Student learning effectiveness is often assessed using rubrics (Fraile et al., 2017). Rubrics are the best tools in assessing the capability of the students to solve mathematical problems (Kamei & Woods, 2016). In addition, using the rubrics is not just a practical assessment instrument. It can also produce more information that needs to be analyzed to give correct feedback with regards to the performance of the students and could quantify the learning outcome (Brookhart, 2017).

One of the major components in the Mathematics Curriculum is problem-solving (Singer et al., 2015). Dealing mathematics with problem-solving can make a context that simulates a real-life, where problem-solving can provide the students with skills in solving problems in daily life (Aydogdu & Ayaz, 2015; Woranetsudathip, 2021). Solving mathematical problems is very important to the general purpose of learning mathematics. It is essential as a human being to solve problems, so each learner needs to have the ability in solving word problems (Rosli, et al., 2013; Woranetsudathip & Yuenyong, 2015). In addition, the National Council of Teachers of Mathematics (NCTM, 2010) stated that problem-solving plays a vital role in the mathematics education of K-12 students.

An Australian educator developed a systematic procedure in analyzing errors committed by the students in solving mathematical problems, which is called the Newman's Error Analysis (NEA). NEA has been lauded worldwide because researchers have utilized it in many places such as Indonesia, Malaysia, Thailand, Australia, etc. (Chusnul et al., 2017). Newman's Error Analysis consists of comprehension, transformation, process skills, and encoding. These four components, comprehension, transformation, process skills, and encoding have different elements. Benjamin Bloom (1956), who created Bloom's Taxonomy, categorized comprehension as one of the levels. According to him, comprehension is composed of interpreting, classifying, explaining, exemplifying, summarizing, and comparing. He also added that process skills involve executing and implementing.
In comparison, Duval (2006) stated that there are two types of transformation: representation and conversion. Whereas in encoding, it's all about writing the answer and labeling (Charles, 1987). These will become the criteria for developing rubrics in solving word problems.

## 2. Literature Review

*2.1 The diversity of mathematical processes and cognitive functioning*

The diversity of mathematical processes and cognitive functioning involved during the mathematical activity often involves understanding the sign (e.g., worded problems, geometric figure, etc.) and the object it signifies. One framework that Duval (2006 exhaustedly offered in understanding the difficulties that many students have in mathematics comprehension is to determine the system of semiotic representations of mathematical objects. These are the cognitive systems required to give access to mathematical objects. One process of gaining such access is through the notion of representation where the process involves accessing individuals' verbal or schematic production through individuals' beliefs, conceptions or misconceptions (Duval, 2006; Fino-Fan et al., 2015). In order to understand how comprehension in mathematics works. It is important to distinguish phenomenological modes of production and the kind of system mobilized for producing any representation. Understanding how the semiotic representation in mathematics differs from other bodies of knowledge such as biology, astronomy, etc., is not found in the concepts. But on the signs or more exactly by how semiotic systems of representation communicate and work with mathematical objects. Since, the representation of signs for another sign is the process involved in processing mathematical objects. In other words in mathematics, signs should not be substituted for an object but rather for another sign. Hence, signs and semiotic representation transformation plays a central part in any mathematical activity.

In the case of word problem-solving, the cognitive distance between the common language (Filipino) of the person, the mathematical language used by the problem (English), and the mathematical use of the language cannot be attributed mainly for the use of a particular vocabulary. In order to successfully perform the needed requirements in the said activity, the doer (in this case the students) need to develop knowledge coordination between the registers of the language, the symbolic expressions of the relations. This coordination is needed in order to perform a successful transformation of the problem and solution.

*2.2 The Use of Scoring Rubric*

Rubric is a popular optimal tool used systematically to evaluate many different skills and subjects (Allen & Knight, 2009). Students can better grasp their work's standards and criteria grades through rubrics and scoring guides (Ragupathi & Lee, 2020). Andrade (2000) emphasized that using scoring rubrics provides students quality inputs that highlight their strengths and weaknesses, which elicits them to deliberate thoroughly on their output. It acts as a tool that determines and communicates expectations to activities given to the students. Rubrics can likewise be considered a discreet scoring sheet utilized exclusively by educators to evaluate students' work fairly, consistently, and efficiently.
When teachers use rubrics, they can openly list assessment criteria that enhance learning, instruction, and assessment (Biggs & Tang, 2007). To achieve a student-centered approach in learning, students can access and co-create necessary criteria in the rubrics (Jonsson, 2014). Moreover, students favor rubrics, particularly instructional rubrics, because they present descriptive feedback on their strengths and areas for improvement. Students agree that using rubrics makes a fairer grading process because they can quickly confirm whether they met the standard. They become more confident and less anxious in working on their tasks (Andrade & Du, 2005).

One way of using difficulties and anxieties commonly encountered by students is to use them as elements of an instructional rubric. These difficulties are identified ahead of time and given to the students for them to be aware of such errors. This role of rubrics plays as an instructional scaffold for students' self-reflection on what to avoid and be guided on what to look for during the problem solving activity. Several factors attributed to difficulties in solving mathematical problems, particularly word problems. For example, Bernardo (1999) studied Filipino-English bilingual students' problem solving skills. Parsing the problem text is one of the difficulties encountered by the students in solving a word problem which is attributed to their proficiency in English. Hence, comprehension is affected by text presentation. In a similar study, Gorgorio and Planas (2001) studied language issues as a crucial component in the process of constructing mathematical knowledge within the classroom. They showed that regardless of communication gap in the language usage and the natural language for mathematics, students' lack of communicative skills can be strengthened by the teachers facilitating the discourse, moving the exploratory to discourse specific talk.

*2.3 The Scoring Rubric as an Assessment Tool*

The increasing emphasis on formative assessment incited a push toward using rubrics in higher education because the criteria's focus relies on the quality of student work (Brookhart, 2013). A rubric is an assessment tool that describes the degree of quality of each criterion from the criteria list of a student's work. It serves as the visual explanation that divides the assignment into parts, reveals patterns, and gives specific descriptions of each characteristic according to varying levels of mastery. Using rubrics strengthens the students' learning process through the process of self-discovery and critical reflection. With continued use, they can quickly point out the recurring problems in their work. Consequently, using rubrics is considered a constructive, student-centered approach to assessment. Rubrics communicate and clarify the educator's expectations to the students' outputs. Thus, these listed expectations prompt a process that boosts student performance (Dawson, 2017).

Ragupathi and Lee (2020) asserted that rubric utilization in higher education helps professors establish a learning-centric and learner-centric environment instead of a task-centric one. It allows them to reflect on the quality of their teaching competence, should there be needs for revision, development, or enhancement. Distinguishing the students' scores from the rubrics effectively addresses most class members' gaps rather than just identifying the individual needs. Furthermore, the same authors added a rubric's important definition based on their professional experiences as professors in Asian universities, including Singapore. They reported that despite the various orientations and ethnolinguistic groups, an effective rubric helped achieve unity in rating students' learning outcomes, allowing for transparency and fairness throughout the assessment process. Using rubrics can develop students' self-efficacy skills because they are immersed in identifying the critical cognitive skills needed to create excellent outputs. When these skills are continuously enhanced, students become independent in planning and self-assessment—thus becoming self-regulated learners because of scoring rubrics (Panadero, 2011).

## 3. Methodology

### *3.1 Research Design*

This research is of a descriptive study that aims to develop a scoring rubric in solving word problems as an assessment tool that could be used to determine the student's level of performance particularly in solving word problems. The first stage in the process of designing the rubrics was to specify the criteria in a manner that could be used to develop a rubric. In developing a scoring rubric Newman's Error Analysis (NEA) was being used as a framework using its four components namely; comprehension, transformation, process skills and encoding. Benjamin Bloom (1956) created Bloom's Taxonomy, one of the levels being categorized was comprehension, according to him, comprehension composed of interpreting, classifying, explaining, exemplifying, summarizing, and comparing. He also added that process skills involve executing and implementing. While Duval (2006) stated that there are two types of transformation these are, representation and conversion. Whereas in encoding, it's all about writing the answer and labeling (Charles 1987). To sum it up, the rubric will have twelve criteria namely; interpreting, classifying, explaining, exemplifying, summarizing, comparing, representation, conversion, executing, implementing, writing the answer, and labeling.

**Table 1: The Original Scoring Rubric in Solving Word Problems with 12 Criteria**

| Criteria | 4 | 3 | 2 | 1 |
|---|---|---|---|---|
| A. Interpreting | Able to interpret the problem which leads to creating a correct equation. | Able to interpret the problem, but gives an incomplete equation. | He/she interprets the problem incorrectly which gives a wrong equation. | No attempt |
| B. Exemplifying | Able to relate correctly and completely to the given problem in the real life situation. | He/she incompletely relates the given problem to the real life situation. | He/she connect the problem to the real life situation incorrectly | No attempt |
| C. Classifying | Able to classify completely and correctly the operations to be used in solving the problem. | He/she classifies incompletely the operations to be used in solving the problem. | He/she classifies incorrectly the operations to be used in solving the problem. | No attempt |

**Table 1 (Cont')**

| Criteria | 4 | 3 | 2 | 1 |
|---|---|---|---|---|
| D. Summarizing | Able to summarize the problem correctly and completely in which he/she can provide the "given" and "what is asked" in the problem. | He/she summarizes the problem incompletely in which he/she provides incomplete "given" and can't identify "what is asked" in the problem. | He/she summarizes the problem incorrectly in which he/she provides wrong "given" and wrong identification of "what is asked" in the problem. | No attempt |
| E. Comparing | Able to compare correctly and completely the numerical values that are present in the problem. | He/she gives an incomplete comparison between the numerical values that are present in the problem. | He/she gives a comparison between the numerical values that are present in the problem incorrectly. | No attempt |
| F. Explaining | Able to explain correctly and completely on how he/she is going to answer the problem. | He/she incompletely explains how to answer the problem. | He/she explains the procedure in answering the problem incorrectly. | No attempt |
| G. Representation | Able to correctly write the word statement to mathematical statements. | He/she writes incompletely the word statement to mathematical statement. | He/she translated the word statement to mathematical statement. | No attempt |
| H. Conversion | Able to convert the final answer to its simplest form. | He/she writes the wrong conversion of the final answer to the simplest form. | He/she doesn't convert the final answer to the simplest form. | No attempt |
| I. Executing | Calculations are correct. All aspects of computations were completely accurate. | Made a minor computational error. Committed one or two errors during computation. | Incorrect computation. Committed three or more errors during computations. | No computation. Give no evidence on how he/she arrived at the answer. |
| J. Implementing | He/she is able to implement the correct process in solving. | He/she made a minor implementation error in which committed only one error. | His/her implementation is incorrect. | No attempt |
| K. Writing the answ | Able to encode or to write the answer correctly and completely. | He/she is able to encode the answer incompletely. | He/she encoded incorrect answers. | No final answer encoded. |
| L. Labelling | Able to encode or to write the correct final answer with a correct label or units. | He/she is able to encode the answer but labeled incorrectly. | He/she is able to encode the answer without a label. | No final answer encoded. |

*3.2 Instruments*

The idea of specifying the rubric was based on a 5-word problem-solving questionnaire used and given to the grade 8 students in a National High School in Northeastern Mindanao. These questions were based on the curricular materials provided by the Department of Education for secondary students. Individual teachers developed their own rubrics to determine the level of problem-solving skills performance of the grade 8 students. Each rubrics were then consolidated by one of the researchers who is also connected to the national high school. Prior to the initial pilot test, the three researchers independently selected items from the developed pool of items that are purported to tap the target construct, concepts and descriptions. When disagreements arise, the decision is based on who has the most salient point on the concepts introduced in the argument.

A try-out group used for PCA consists of 240 randomly selected freshman to senior teacher education students enrolled in a public higher education institutions in the Northeastern Mindanao Caraga region who were utilized for the validation of the initial concepts included in the rubrics. The number of samples used follows Comrey and Lee (2013) ratio of N:10, where N is the number of concepts.

The final respondents used for EFA were the 154 teachers with varied teaching experiences ranging from those newly hired to teach mathematics to seasoned teachers with almost 15 years' experience in the service. They are the randomly selected from the Eastern Zone Secondary Schools of Agusan del Norte Division, namely; Jagupit National High School, Jaliobong National High School, Kitcharao National High School, and Santiago National High School and 38 grade 8 students (12 females and 25 males) of Kitcharao National High School of Agusan del Norte Division.

**Table 2: The Research Questionnaire**

Direction: Using a scale of 1= Not important to 7= very important, please rate the following criteria in solving word problem

| Criteria | Rate |
|---|---|
| A. Interpreting | |
| B. Exemplifying | |
| C. Classifying | |
| D. Summarizing | |
| E. Comparing | |
| F. Explaining | |
| G. Representation | |
| H. Conversion | |
| I. Executing | |
| J. Implementing | |
| K. Writing the answer | |
| L. Labelling | |

*3.3 Technique of Data Analysis*

After identifying the twelve criteria, these have been used in the research questionnaire so that the respondents (teachers) could evaluate each criterion. The research questionnaire is a seven-point Likert-like scale designed to solicit an individual's view to rate from 1= not important up to 7 = Very Important. The researcher used exploratory factor analysis in analyzing and reducing the criteria for the development of revised word problems using the method suggested by Matsunaga (2010). This approach utilizes (a) an initial set of items that are first screened by PCA, (b) the remaining items are subjected to EFA, and (c) an extracted final factor solution. Moreover, factor analysis has its origin in the early 1990's which Charles Spearman's interest (Harman, 1976) and demonstrate how to collapse the information compiled in a task oriented checklist rubric into more efficient set of parsimonious performance number of criteria (Baryla et al., 2012). Since the scoring rubric was already developed and utilized by the teachers based on individual perception of components to be included. After collection of items it was pilot tested to prospective teachers enrolled in a public higher education institution and validated through PCA. After the initial results were obtained it was then employed by different mathematics teachers with varied years of experience teaching mathematics at Eastern Zone Secondary Schools of Agusan del Norte Division to assess the problem solving questionnaire skills of the students in order to determine the student's level of performance at the same time to check and test the revised rubric. Lastly, the instrument was pilot tested by the three experts of Kitcharao National High School to assess the problem solving questionnaire answered by the students in order to determine the student's level of performance at the same time to check and test the revised rubric.

## 4. Findings and Discussion

The data consists of 12 criteria in scoring a rubric in solving word problems, evaluated by 154 teachers. To determine how many significant factors there are, both Kaiser's (1960) eigenvalue and scree test are used. In order to proceed with the analysis using Exploratory Factor Analytic technique (Hair et al., 2010), the following should be done as ad-hoc or assumptions.

**Table 3: Ad-hoc Measures for Factor Analysis**

| | | |
|---|---|---|
| Kaiser-Meyer-Olkin Measure of Sampling Adequacy. | | .887 |
| Bartlett's Test of Sphericity | Approx. Chi-Square | 1914.935 |
| | Df | 66 |
| | Sig. | 0.000 |

It can be gleaned from Table 3 the Kaiser Meyer Olkin (KMO) and Bartlett's test. Two of the most commonly used tests before doing (ad-hoc) factor analysis. Kaiser Meyer Olkin (KMO) measures the sampling adequacy to assess the appropriateness of using factor analysis on the data set, while Bartlett's test which measures the strength of relationship among the variables. Kaiser (1974) recommended 0.5 (value for KMO) as minimum (barely accepted). Furthermore, Barlett's test and KMO measures were ($p<0.01$) and 0.887, respectively. This shows that there are no issues for the ad-hoc measures for exploratory factor analysis, hence factor analysis is suitable for the study.

**Table 4: Component extraction**

| Component | Initial Eigenvalues | | |
|---|---|---|---|
| | Total | % of Variance | Cumulative % |
| 1 | 7.408 | 61.737 | 61.737 |
| 2 | 2.089 | 17.409 | 79.147 |

Eigenvalue reflects the number of extracted factors. As the data revealed in Table 4, the first factor accounts for **61.737%** of the variance and the second factor is **17.409%.** All remaining factors are not significant. So only two factors have been retained, since according to the principle of parsimony that researchers should strive for simple measurement and states that simple criterion leads easy to score (Raykov & Marcoulides, 1999). Moreover, the percentage of variance shared by component 3 is just **4.603,** the gap between the 2 and 3 components is relatively wide. So the decision to choose the 2 components supports the suggestion of Park et al., (2002) where the number of components retained is grounded on the conceptualization of target construct, that is, the principle of parsimony and a NEA. Specifying too many components makes makes it difficult to implement especially if the purpose of the instrument is for instructional purposes.

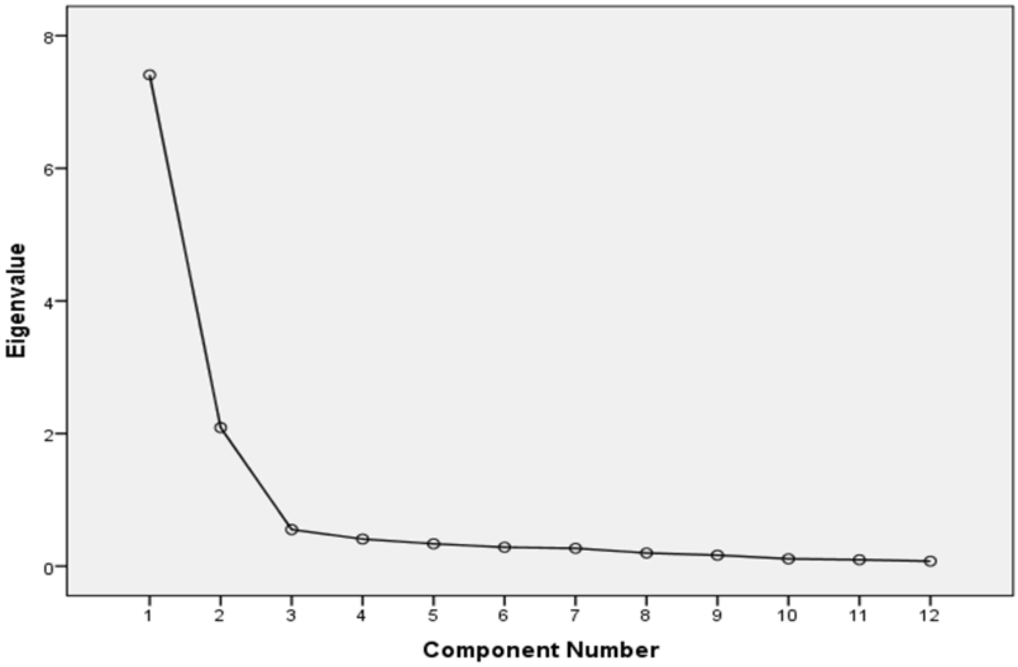


**Figure 1: The Eigen Values of Factors**

The scree plot showed the graph of eigenvalues against all factors and it determines the number of factors to be extracted. The point of interest in using a scree plot here is to determine where the curve starts to flatten. Moreover, the distance on the succeeding consecutive number of components is no longer differentiable when projected by a horizontal line. It can be seen in Figure 1 that the curve begins to flatten between 3 and 4. Note also that factor 3 onwards have an eigenvalue of less than 1, so either two or three factors have to be retained. Only 2 components were used in the study since there is a wide gap between the 2 and 3 components. The table 5 displays that the variables **Executing, Implementing, Interpreting, Labelling, Representation, Writing the answer** are loaded in factor 1, meanwhile **Conversion** is substantially loaded on factor 1 and factor 2 (cross-loading). All the remaining variables are substantially loaded on factor 2 (Table 5).

According to Osborne et al. (2008) that after the rotation, we have to compare the item loading tables; the one with the "clearest" factor structure with few item cross-loadings and no factors with fewer than three items, has the best fit to the data. Obviously the item loading tables with two factors will be carried out.

The result of factor analysis suggests that the rubric needs a modification, even if the researcher is trying to measure more competencies with those 12 criteria it is likely to have only two components to be measured in which redundancy and do not supply any additional information.

**Table 5: Oblique Rotated Component Matrix solution**

| | Component | |
|---|---|---|
| | 1 | 2 |
| Writing the answer | .909 | |
| Executing | .898 | |
| Implementing | .877 | |
| Interpreting | .838 | |
| Labelling | .752 | |
| Representation | .745 | |
| Comparing | | .937 |
| Exemplifying | | .882 |
| Classifying | | .857 |
| Explaining | | .831 |
| Summarizing | | .779 |
| Conversion | .523 | .656 |

Matsunaga (2010) suggested that once factors are generated using EFA some of them would not contribute appreciably to account for data's variance. One way of identifying the loaded items in a component is to specify the rotation. Promax rotation was used since it operates to obtain the solution raise the factor loadings to a stated power so that the resultant factors/components are maximally distinguishable (Comrey & Lee, 2013). The purpose of choosing this rotation is allow for a natural pattern to emerge and not a result of researchers' choice. Only the conversion item cross-load to both components. One widely utilized approach in choosing component where to place the cross-loaded item is to focus on the highest loading or a judgement-call on the part of the researcher depending on the priori criteria set or theoretical framework anchored (Hair et al., 2019). Based on the observations the common thing about Interpreting, Representation, Executing, Implementing, Writing the answer and Labelling is that they are a procedure of showing the solutions in answering the problem. Whereas, Summarizing, Comparing, Exemplifying, Classifying and Explaining involves understanding the given problem and focusing on "knowing or identifying something". That's why they are loaded at two different constructs. The original rubric, which is composed of 12 criteria, should collapse into two performance criteria. Through EFA, it was suggested that these criteria (Comparing, Exemplifying, Classifying, Explaining and Summarizing) should be loaded on the same construct. Declarative Knowledge, is about knowing "what" which refers to the representations of objects and information that a person knows. Like, knowing what needs to be solved, identifying the given and what is asked in the problem, etc. The other construct is composed of Interpreting, Representation, Executing, Implementing, Writing the answer, Labelling and Conversion can be called as Procedural Knowledge because it involves a certain step, process or procedure in solving word problems (Salaberry, 2018).

**Table 6: Revision of Scoring Rubric in Solving Word Problems based on Factor Analysis Results.**

| Criteria | Score level and description | | | | |
|---|---|---|---|---|---|
| | 0 | 1 Point | 2 Points | 3 Points | 4 Points |
| Declarative Knowledge | • No response<br>• No attempt | He/she incorrectly identifies both the "given and what is asked in the problem". | He/she correctly identifies the "given" but incorrectly determines "what is asked in the problem" (or the other way around) | He/she identifies both the given and what is asked in the problem but with incomplete data either the given or on what is required or to both | He/she correctly identifies both the "given and what is asked in the problem" |
| Procedural Knowledge | • No final answer encoded<br>• No attempt | Encoded a wrong answer without a solution or with a wrong solution | Encoded a correct mathematical representation of the problem but with wrong solution and answer | Encoded a correct mathematical representation of the problem and solution but with minor errors in the final answer (minor error refers to the error in simplification or performing the last operation to get the final answer) | Encoded a correct answer with complete/correct solution |

One of the expert's remarks about the developed rubric is that, the scoring rubric has a great help in assigning corresponding points of a student's answer particularly in solving word problems. Wrapping those 12 criteria into a smaller number of performance criteria which measures specific learning competencies, gives the teacher more accuracy in determining the student's level of performance. Moreover, the revised rubric is less burdensome or more efficient to use. This is supported by the idea of Baryla, Shelley and Trainor (2012), they concluded that "with fewer criteria to evaluate, the quality of the data will likely improve and the improved rubrics then will provide higher quality information concerning achievement of learning objectives while reducing the overall burden of teacher conducting the assessment".

**Table 7: The Student's Performance in Solving Word Problems.**

| **Student's Problem Solving Scores** | | **f(percent)** |
|---|---|---|
| Above Average | 38-40 | 7 (18.92) |
| Average | 30-37 | 26 (70.27) |
| Below Average | 22-29 | 4 (10.81) |
| | 21- below | 0 |

It can be observed in Table 7 that in terms of the performance of the students in solving word problems 70.27 % or 26 students fall under the rank of 30-37, which means that majority is in Average level. Moreover, 18.92% or 7 students performed on Above Average in the said problem solving performance. In addition, 10.81% or 4 amongst students has Below Average. It implies that the students from this section are performing well in solving word problems, since only 10.81 % are below average and it could be determined with the use of this scoring rubric in solving word problems.

## 5. Conclusion

Competencies employed in the teaching of secondary mathematics are set by the Department of Education, using systems approach. The developed competencies are brought down from the top (curriculum developers) to the bottom implementers (teachers). In the implementation process, understanding common grounds of sources of difficulties and success stories is important. Instead of individuality, the method in this study employed a collective approach in developing different constructs that serve as a basis in measuring specific learning competencies. After using the revised rubric, it was found out that the student's performance was on average level. The revised rubric can thoroughly assess the student's performance specifically their problem solving skills. Methods employed using exploratory factor analysis in developing scoring rubric in solving word problems help teachers systematize a method of organizing constructs. Out of twelve criteria, only two criteria emerged, which is the basis for a scoring rubric. The transferability of using the developed questionnaire in a different worded problem situation may affect the outcomes if there is considerable cognitive distance between the natural language and the mathematical use of the language where the instrument was based. The preparation gap of the different groups of participants (prospective teachers and in-service teachers) involved during screening and finalizing may also affect the resulting rubric. Experience relates to an increase in domain-specific efficacy as a teacher went through preparation and teaching practice to becoming a novice and then a more experienced teacher (Chan, 2008). Teaching mathematics, however, MacCallum et al. (1999) warned that sample size issues in using N: 10, where it stands for ratio of variables to samples, may pose a risk of overestimating the resulting components without considering a parallel analysis and confirmatory factor analysis.

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