Asia Research Network Journal of Education
Vol. 3, No. 1, pp. 13-24, Jan. – Apr. 2023

Enhancing Grade 10 Students' Problem Solving Ability in Basic Knowledge on Analytical Geometry Flipped Classroom

Ketsaraporn Suanse*
Khon Kaen University Demonstration Secondary School,
Khon Kaen, Thailand

Chokchai Yuenyong
Faculty of Education, Khon Kaen University, Thailand
*Corresponding author email: sketsa@kku.ac.th

Received: 13 Jan 2023 Revised: 20 Apr 2023 Accepted: 21 Apr 2023

Abstract: The paper aimed to examine Grade 10 students' problem solving in basic knowledge on analytical geometry through the analytic geometry flipped classroom. The participants included 36 Grade 10 students who were studying in geometry flipped classroom at Khon Kaen University Demonstration School in academic year of 2019. Methodology regarded mixed methods. The intervention provided two weeks of the basic knowledge on analytical geometry flipped classroom that was one of three sub-unit of the analytic geometry flipped classroom. Students' problem solving in basic knowledge on analytical geometry was collected when they performed on three tasks of the PBAG. The PBAG students' responses were analyzed based the problem-solving scoring rubric that adapted from Charles et.al. (1987). The finding revealed that majority of students held excellent level of holistic assessment for problem solving on analytic geometry. Most of students held the excellent problem-solving ability for three tasks. These included 1) finding the distance between two points and find the center of segment line, 2) finding the area of the given polygon, and finding the distance between two points and find the center of segment line, and 3) finding the midpoint, the projection of the set point, distance between two points and center the segment line.
Keywords: analytic geometry, flipped classroom, problem solving

1. Introduction

Two pillars of mathematics are algebra and geometry (Atiyah, 2001). And, literatures argued about what geometry to include in the school curriculum have been going on for at least 100 years (Sinclair 2008; Usiskin 1987). The question about what and how to teach geometry has been raised for long. Jones and Fujita (2013) analyzed how the features of geometry and approaches to geometry appear in the sampled textbooks from major publishers in England and Japan. They argued that geometry curriculum related to the teaching of reasoning and proof, and the teaching of problem solving. Moriotti and Balacheff (2008) suggested that geometry can give other learning possibilities, such as problem-solving and mathematical modeling, which are very important in the teaching and

learning process. Geometry teaching of problem-solving, which is historically and internationally recognized as one of the most difficult components of mathematics education research (Setthaphongsakorn and Yuenyong, 2019; Villani et. al., 1994). The NCTM (1991) prepared the geometry textbook that considered geometry from multiple perspectives. To do so, students need to be engaged by different activities that aimed to create collaborative environment for problem solving. Students would work in small groups, discuss their findings in class as a whole group, and enhance their metacognitive abilities while problem-solving (Ebal et.al., 2019; Jackaria et.al., 2019; Woranetsudathip et.al., 2021).

According to Schoenfeld (1985), in order to solve problems successfully, one must be equipped with and properly use relevant resources (e.g. mathematics concepts and procedures), heuristic strategies (specific and general heuristics), metacognitive control (monitoring and overseeing the entire problem solving process), and appropriate beliefs (one's perspective, motivation, and confidence). Similarly, Mayer and Wittrock (2006) stated that students require five types of knowledge in order to be competent problem solvers. These included facts (knowledge about characteristics of elements), concepts (knowledge of categories, principles, or models), strategies (knowledge of general methods), procedures (knowledge of specific procedures), beliefs (cognitions about one's problem solving competence or about the nature of problem solving), meta-cognitive knowledge (awareness and control of one's own cognitive processing and includes beliefs). And, in order to prepare a practical idea for teachers to assess students' problem solving ability, we need some framework of problem solving assessment. Regarding on Charles et.al. (1987), ability of mathematics problem solving was evaluated into 4 dimensions. These included 1) understanding mathematical problem, 2) selecting choices of strategies of problem solving, 3) finding the right solutions, and 4) conclusion for the right answers.

The flipped classroom is appropriate pedagogy for problem solving. Jonathan Bergmann and Aaron Sams, two chemistry professors from Woodland Park, Colorado, popularized the flipped classroom among educational practitioners in 2007. Bergmann and Sams (2012) argued that, rather than receiving a lecture, it is possible to engage all students in learning, regardless of content area or individual differences among students, by asking students to watch videos of lectures prior to coming to class and then spending the time in class discussing assignments. During class, they could involve students in problem-solving and laboratory investigations, thus "flipping" the classroom (Cheng et.al., 2019). Additionally, flipped classroom is defined by Bishop and Verleger (2013) as "a new pedagogical method that employs asynchronous video lectures and practice problems as homework, as well as active, group-based problem-solving activities in the classroom." They eliminated implementations that did not use videos outside of the classroom. They argued that if the definition is too wide and does not confine instruction to videos, the flipped classroom would include "assigning reading outside of class and having discussions in class."

Analytic geometry's core notions are the simplest geometric elements (points, straight lines, planes, second-order curves, and surfaces). The methods of coordinates and elementary algebra are the primary tools of study in analytic geometry. The study of geometry using coordinate points is known as coordinate geometry (or analytic geometry). It is possible to find the distance between two points, divide lines in a m:n ratio, identify the midpoint of a line, calculate the area of a triangle in the Cartesian plane, and so on using coordinate geometry. Certain terms in Cartesian geometry should be properly understood (Amadeo, 2018). Regarding on the concept of analytical geometry, teaching and learning for analytical geometry require students to successfully solve problems solving. The flipped classroom would allow teachers more chances to foster students to solve problems through working together in the classroom.

2. Methodology

Methodology regarded interpretive paradigm. Students' problem solving ability was interpreted through what they performed about problem solving on the basic knowledge about analytical geometry in the flipped classroom.

2.1 Participants

Participants included 36 Grade 10 students who were studying in geometry flipped classroom at Khon Kaen University Demonstration School (Modindaeng) in academic year of 2019.

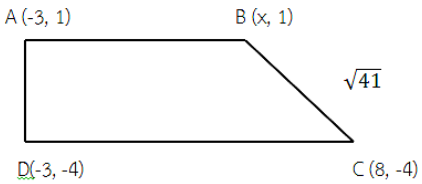
2.2 Intervention of the basic knowledge on analytical geometry flipped classroom

The flipped classroom of the basic knowledge on analytical geometry was designed students to learn about theories via online classroom and learn how to solve problem in person in classroom practicing. Details of the intervention have been clarified separately in another paper (Suanse and Yuenyong, 2021). The online learning activities were designed via the Google classroom. These online activities provide theories and mathematical exercise for the following objectives:

1) find the projection of the set point,
2) find the distance between two points and find the center of the segment line,
3) find the area of the given polygon,
4) apply knowledge about the midpoint and the dividing point to solve problems,
5) find the intersection of the median.

The classroom practicing provided students to solve the mathematical problems of the basic knowledge on analytical geometry (PBAG) with supporting by teachers. These problems included:

- Task 1: How far is the midpoint between point A(3,5) and point B(-7,1) from point C(-1,4)?
- Task 2: According to the given figure, □ ABCD has points A(-3, 1), B(x, 1) , C (8, -4) and points D(-3, -4). Find the area □ ABCD when BC = $\sqrt{41}$ units



- Task 3: Let point A (-2, 2), B (3, -8). Let C be the upper point $\overline{AB}$ such that AC : CB = 3 : 2. And, P be the projection of point C on line y = 4. Find the distance between point C and point P.

The online activities and classroom practicing, therefore, allowed students to study the lessons in advance and the program helps learners to have more time working and practicing learning activities to solve Mathematics problems in actual classrooms. Teaches works as mentors and experts in organizing learning activities.

2.3 Data collection and analysis

Students' problem solving in basic knowledge on analytical geometry was collected when they performed on three tasks of the PBAG. The PBAG students' responses were analyzed based the problem-solving scoring rubric that adapted from Charles et.al. (1987). Regarding on Charles et.al. (1987), ability of mathematics problem solving was evaluated into 4 dimensions. These included 1) understanding mathematical problem, 2) selecting choices of strategies of problem solving, 3) finding the right solutions, and 4) conclusion for the right answers. The scoring rubric could be showed in the table 1. Then, score of

level in each dimension of problem solving will be summed in order to assess ability of students' problem solving on each task through holistic scale as showed in the table 2. However, same score of holistic scale would not represent the same ability because same score of holistic scale probably did not come from the score level in each dimension of problem solving. The ability of students' problem solving, therefore, will be also clarified the trace of holistic score. To communicate trace of dimension level for holistic score, each dimension of problem solving will be represented by the following abbreviation: U – understanding mathematical problem, S - selecting choices of strategies of problem solving, F - finding the right solutions, and C - conclusion for the right answers

Table 1: the problem-solving scoring rubric (adapted from Charles et.al., 1987)

Dimensions	level	criteria
U - understanding mathematical problem	2	Students present clue of their understanding problems (e.g., students write *their own words* about crucial issues of problems, students underline keywords to represent the problems). And, students identify correctly facts of problem situations.
	1	Students present clue of their understanding problems (e.g., students write the word about crucial issues of problems, students underline keywords to represent the problems). However, students identify partially facts of problem situations.
	0	Students could not present the clue of understanding problems and could not present some facts of problems.
S - selecting choices of strategies of problem solving	2	Students select the right strategies for successfully solving problem such as drawing pictures, drawing diagraph, or drawing formula, or backward thinking in order to provide logical reasons, prediction, checking and so on.
	1	Students show traces of selecting the strategies but students could not successfully solve problem.
	0	Students could not show any traces of selecting the strategies of problem solving.
F - finding the right solutions	2	Students show traces of their solving problem (e.g., clarify correct facts related to the problems). And, those clues could allow them to find the right solutions.
	1	Students show some traces of their solving problem but those traces could suggest students to only find partial of right solutions. Or, students could not show traces of their solving problem but they could find the right answers.
	0	Students could not show traces of their solving problem. And, they could not find any correct answers.
C - conclusion for the right answers	2	Students could draw conclusion for completely right answers.
	1	Students could draw conclusion for partial of right answers. Or, students identify the wrong answers but they show some traces of solving problems that could find what and how they mistook.
	0	Students could not draw conclusion for answer.

Table 2: holistic scale of students' problem-solving ability

Holistic scale	Students' problem-solving ability
7-8	Excellence
5-6	Good
3-4	Fair
0-2	Watch list

3. Findings and Discussion

Students' problem solving in basic knowledge on analytical geometry could be clarified regarding on what they performed on solving the problem on task1 – 3. It found that majority of students held the excellent problem-solving ability for three tasks.

3.1 Students' problem solving for finding the distance between two points and find the center of segment line

Interpreting what they solve the problem of task 1 could assess students' ability of problem solving for finding the distance between two points and find the center of segment line. To solve task 1, students need to do two steps including: 1) students must first find the midpoint between two points using their knowledge of the midpoint of a line segment, and 2) students must find the distance between the midpoint obtained in step 1 and point C (-1,4) by using their knowledge of finding the distance between two points. The result of students' ability of problem solving could be summarized as showed in the table 3.

Table 3: Students' ability of problem solving for finding the distance between two points and find the center of segment line

Students' problem-solving ability	Holistic score	Trace of dimension level for holistic score				Frequency of students	Percents of frequency
		U	S	F	C		
Excellence	8	2	2	2	2	13	36.11
	7	1	2	2	2	6	16.67
	7	2	2	2	1	4	11.11
Good	6	2	2	1	1	2	5.55
	6	1	2	2	1	1	2.78
	5	1	2	1	1	2	5.55
	5	2	2	1	0	1	2.78
Fair	4	2	1	1	0	2	5.55
	4	1	1	2	0	1	2.78
	3	2	1	0	0	1	2.78
Watch list	2	0	1	1	0	1	2.78
	2	1	1	0	0	1	2.78
	1	0	0	0	1	1	2.78
					Total	36	100.00

According to the table 3, majority of students (approximately 64%) held excellent ability of problem solving for finding the distance between two points and find the center of segment line. Interestingly, some students, who held excellent ability, seems to has difficulty on understanding mathematical problem (6 students got level 1 on U) and conclusion for the right answers (4 students got level 1 on C). However, some of students held low ability of problem solving (fair and watch list). Commonly; good, fair and watch list ability students seem to has difficulty on conclusion for the right answers. Examples of how we interpreted students' ability could be clarify as below:

Example of excellent ability student

The figure 1 showed performance of excellent ability student who got 8 in holistic score of problem solving for finding the distance between two points and find the center of segment line. And, trace of dimension level for holistic score could be clarified as following. He or she got level 2 on U - understanding mathematical problem because of two reasons. First, student could write that the problem has given the coordinates of points A, B, and C. Second, student could identify what the problem asked when he or she mentioned that "the distance from the middle point of line segment AB to point C, along with drawing a diagram for understanding." He or she got level 2 on S - selecting choices

of strategies of problem solving because he or she could find two strategies of solving problem. First strategy was the splitting the case of finding a solution in order to make the problem more concrete. This would make a difficult problem becomes simpler and in a familiar form so that the answer to the problem can be found. Second strategy was the selecting or formulating relevant formulas to find the correct answers. This could be seen when he or she was solving the problem by using the formula to find the midpoint of line segment AB and then use the formula to find the distance between two points. And, as earlier clarifying showed that he or she could show traces of their solving problem and draw conclusion for completely right answers. Therefore, he or she got level 2 on F and C.

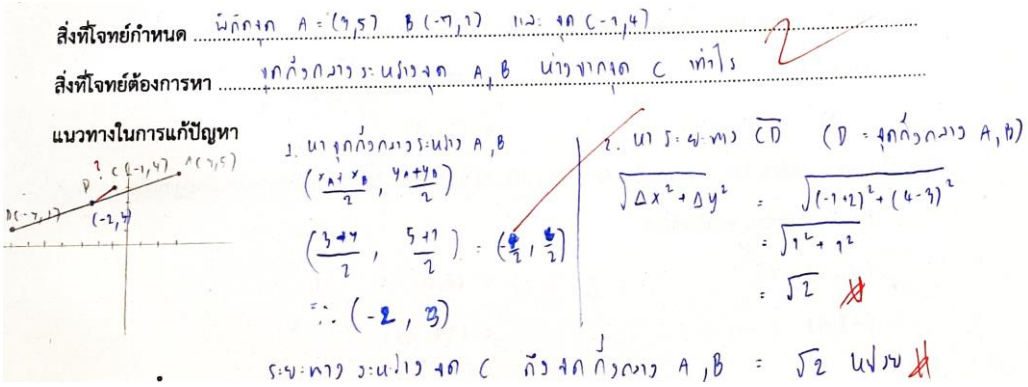


Figure 1: performance of excellent ability student who was solving about finding the distance between two points and find the center of segment line

Example of good ability student

The figure 2 showed performance of good ability student who got 5 in holistic score of problem solving for finding the distance between two points and find the center of segment line. And, trace of dimension level for holistic score could be clarified as following. He or she got level 2 on U - understanding mathematical problem because of two reasons. First, student could write that the problem has given the coordinates of points A, B, and C. Second, student could identify what the problem asked when he or she mentioned that "the distance from the middle point of line segment AB to point C, along with drawing a diagram for understanding." He or she got level 2 on S - selecting choices of strategies of problem solving because he or she could find two strategies of solving problem. First strategy was the splitting the case of finding a solution in order to make the problem more concrete. This would make a difficult problem becomes simpler and in a familiar form so that the answer to the problem can be found. Second strategy was the selecting or formulating relevant formulas to find the correct answers. This could be seen when he or she was solving the problem by using the formula to find the midpoint of line segment AB and then use the formula to find the distance between two points. And, as earlier clarifying showed that he or she could show traces of their solving problem and draw conclusion for completely right answers. Therefore, he or she got level 2 on F and C.

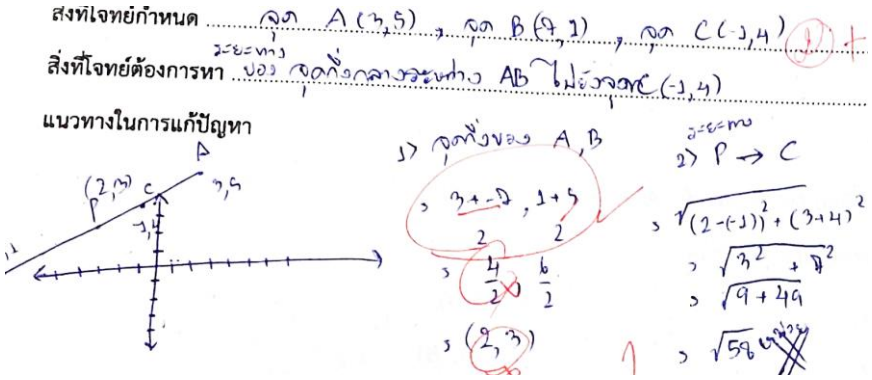


Figure 2: performance of good ability student who was solving about finding the distance between two points and find the center of segment line

Example of fair ability student

The figure 3 showed performance of good ability student who got 3 in holistic score of problem solving for finding the distance between two points and find the center of segment line. And, trace of dimension level for holistic score could be clarified as following. He or she could identify what the problem asked. This could be seen when he or she mentioned that "the distance from the midpoint of line segment AB to point C"; however, he or she could not draw the picture to represent what the problem asked. He or she could not identify the strategy of problem solving, and could not find the correct answers, then.

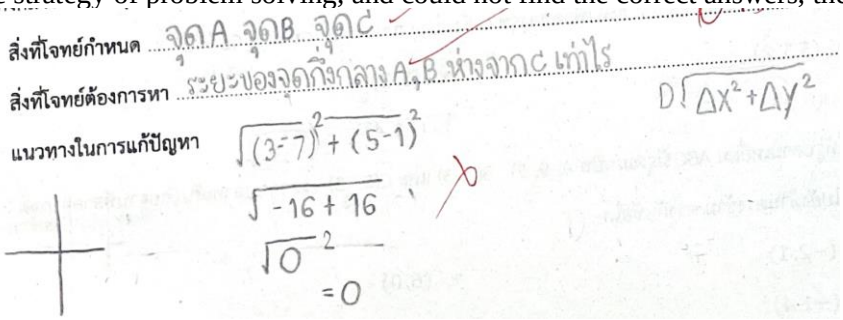


Figure 3: performance of fair ability student who was solving about finding the distance between two points and find the center of segment line

3.2 Students' problem solving for finding the area of the given polygon, and finding the distance between two points and find the center of segment line

Interpreting what they solve the problem of task 2 could assess students' ability of problem solving for finding the area of the given polygon, and finding the distance between two points and find the center of segment line. To solve task 2, students need to do two steps including: 1) x must first be found, which is the reverse of how to find the distance between two points, and 2) In case of knowing vertex, find the area of ABCD quadrilateral using knowledge of finding the area of polygons. The result of students' ability of problem solving could be summarized as showed in the table 4.

Table 4: Students' ability of problem solving for finding the area of the given polygon, and finding the distance between two points and find the center of segment line

Students' problem-solving ability	Holistic score	Trace of dimension level for holistic score				Frequency of students	Percents of frequency
		U	S	F	C		
Excellence	8	2	2	2	2	10	27.77
	7	2	2	2	1	12	33.33
Good	6	2	2	1	1	3	8.33
	6	1	2	2	1	1	2.78
	5	2	2	1	0	1	2.78
	5	1	2	1	1	1	2.78
Fair	4	2	1	1	0	3	8.33
	4	2	2	0	0	1	2.78
	4	2	1	0	1	1	2.78
	4	1	2	1	0	1	2.78
	3	2	1	0	0	1	2.78
Watch list	2	1	1	0	0	1	2.78
					Total	36	100.00

According to the table 4, majority of students (approximately 61%) held excellent ability of problem solving for finding the area of the given polygon, and finding the distance between two points and find the center of segment line. However, some of students held low ability of problem solving (fair and watch list). Commonly; fair and watch list ability students seem to has difficulty on conclusion for the right answers. Examples of how we interpreted students' ability could be clarify as below:

Example of excellent ability student

The figure 4 showed performance of excellent ability student who got 8 in holistic score of problem solving for finding the area of the given polygon, and finding the distance between two points and find the center of segment line. And, trace of dimension level for holistic score could be clarified as following. He or she showed his or her understanding mathematical problem when student identified the coordinates of the 4 corner points of the polygon ABCD, length of side BC as $\sqrt{41}$ unit, and need to find area of ABCD. He or she could draw the conclusion for correct answer. He or she could find the strategies of problem solving. Three strategies were mentioned:

1) Breaking down the sub-cases of finding a solution to visualize complex problems becomes simpler and familiar;

2) Backward thinking, it starts from the final information, then think back step by step to the initial information;

3) Selecting or defining relevant formulas used to solve problems. This could be seen when he or she explain 2 steps of finding the answer. Firstly, identify the coordinate point B with unknown variable is B (x, 1) and $\sqrt{41}$ unit of length of side BC. Then, thinking backwards to find x using the formula for the distance between two points yields two x values. And, then select only one value of x with providing of logical reason. Secondly, find the area ABCD using the polygon area formula.

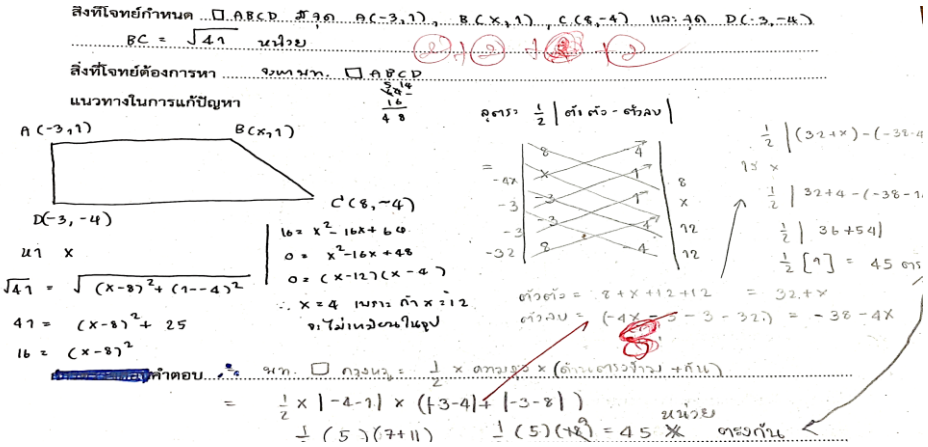


Figure 4: performance of excellent ability student who was solving about finding the area of the given polygon, and finding the distance between two points and find the center of segment line

Example of good ability student

The figure 5 showed performance of good ability student who got 6 in holistic score of problem solving for finding the area of the given polygon, and finding the distance between two points and find the center of segment line. And, trace of dimension level for holistic score could be clarified as following. The figure 5 showed what his or her performance of U – understanding mathematical problem and S - selecting choices of strategies of problem solving as same as the figure 4. However, they got only level 1 for F - finding the right solutions, and C - conclusion for the right answers because they lack of backward thinking for finding value of x and then they could not draw the correct answer. The could be seen what they did on the way of finding the right solutions, he or she used the distance between two points formula to find the value of x. Then, they got two values of x, 12 and – 12, which are invalid due to errors in solving the equation in absolute terms. And, they did not think backwards even though the obtained x value contradicts the given picture of ABCD area.

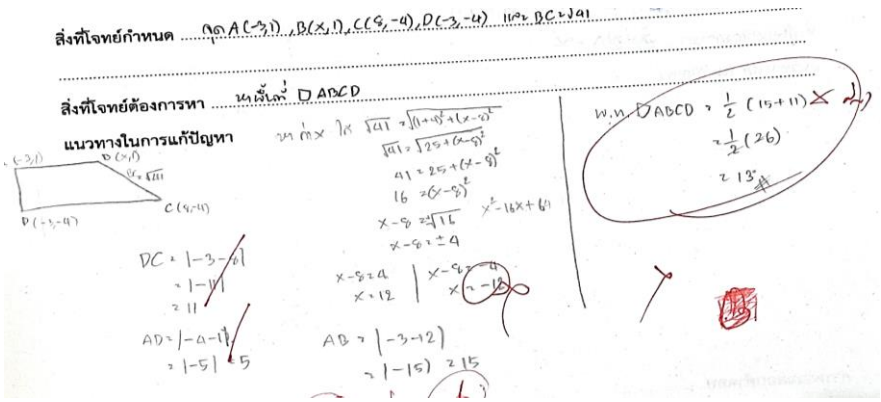


Figure 5: performance of good ability student who was solving about finding the area of the given polygon, and finding the distance between two points and find the center of segment line

3.3 Students' problem solving for finding the midpoint, the projection of the set point, distance between two points and center the segment line

Task 3 engaged students to solve problem about 1) apply knowledge about the midpoint and the dividing point to solve problems, 2) find the projection of the set point, and 3) find the distance between two points and find the center of the segment line. To solve task 3, students probably work into three steps of problem solving. These include:

- Find the coordinates of point C using the knowledge of dividing line segments.
- Find the coordinates of point P, which is the projection of point C on the line y = 4, using knowledge of the projection of any point on the line.
- Find the distance between point C and point P using the knowledge of finding the distance between two points.

The result of students' ability of problem solving in task 3 could be summarized as showed in the table 5.

Table 5: Students' ability of problem solving for finding the midpoint, the projection of the set point, distance between two points and center the segment line

Students' problem-solving ability	Holistic score	Trace of dimension level for holistic score				Frequency of students	Percents of frequency
		U	S	F	C		
Excellence	8	2	2	2	2	16	44.44
	7	2	2	2	1	4	11.10
Good	6	2	2	1	1	2	5.56
	6	2	2	2	0	2	5.56
	6	1	2	2	1	1	2.78
	5	2	2	1	0	4	11.10
Fair	4	2	2	0	0	1	2.78
	3	2	1	0	0	1	2.78
Watch list	2	2	0	0	0	2	5.56
	2	0	2	0	0	1	2.78
	1	1	0	0	0	1	2.78
	0	0	0	0	0	1	2.78
					Total	36	100.00

According to the table 5, majority of students (approximately 55%) held excellent ability of problem solving for finding the midpoint, the projection of the set point, distance between two points and center the segment line. Number of students (approximately 25%) held good ability of problem solving. And, there were various traces of problem solving for good ability students. However, some of students held low ability of problem solving (fair and watch list). Commonly; fair and watch list ability students seems to has difficulty on finding strategies for problem solving, finding right solutions, and conclusion for the right answers. Example of how we interpreted students' ability could be clarify as below:

Example of good ability student

The figure 6 showed that a student performed into good ability of problem solving for finding the midpoint, the projection of the set point, distance between two points and center the segment line. He or she got 6 in holistic score of problem solving. Trace of problem solving could be clarified as level 2 for U and S, and level 1 for F and C. U was evaluated into level 2 because he or she identified for the following issues:

- point coordinates of line segment AB are points A, B;
- point C is divided with the ratio AC : CB = 3 : 2;

- P is the projection of point C on the line y = 4; and
- the problem asked about the distance between point C and point P.

S was evaluated into level 2 because he or she identified three strategies for problem solving. These included:

- Drawing the picture to mention what the problem asked, to clarify relation among data, and to illustrate the possible strategies of problem solving.
- Breaking down the sub-cases of finding a solution to visualize complex problems becomes simpler and familiar.
- Selecting or defining relevant formulas used to solve problems. This could be seen when he or she explain 3 steps of finding the answer. Firstly, find the coordinates of the intersection point C by using the line segment intersection formula. Secondly, find the projection coordinates of point C on the straight line. y = 4 is the point P(0, 4) is the value obtained from drawing. This is an invalid value due to the error of drawing incorrectly. Thirdly, write a method to find the distance between point C and point P using the distance formula, but the answer is incorrect because the coordinates of point P obtained in step 2 are incorrect.

And, the F and C were evaluated into level 1 because student mistook on some steps of solve formulas. He or she, then, could not completely find the right solutions, and conclude the right answers.

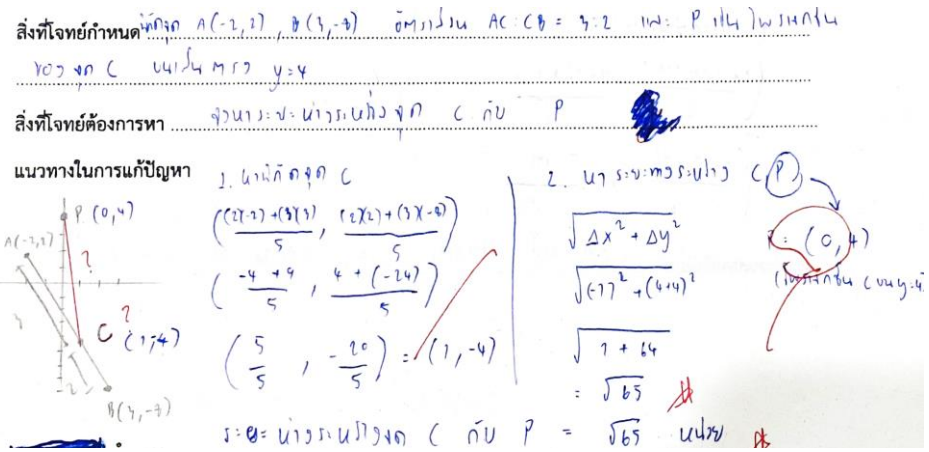


Figure 6: performance of good ability student who was solving about finding the midpoint, the projection of the set point, distance between two points and center the segment line

4. Conclusion

It indicated that the flipped classroom of the basic knowledge on analytical geometry could develop students' problem solving ability. It could be discussed that the flipped classroom allow teacher more chance to foster of solving the problem in classroom. Most of students held the excellent problem-solving ability for three tasks. These included 1) finding the distance between two points and find the center of segment line, 2) finding the area of the given polygon, and finding the distance between two points and find the center of segment line, and 3) finding the midpoint, the projection of the set point, distance between two points and center the segment line.

A conceptual scoring rubric, adapted from Charles et al**.** (1987), could assess students**'** levels of success in solving problems**.** The scoring rubric could explain students**'** problem-solving actions matched the characteristics of good problem solvers such as the ability to use diverse strategies, flexibility approaches, verification actions, and the ability to deal

with irrelevant information in problem tasks**.** Students frequently reflect on the mathematical ideas in the assignments during problem-solving activities, producing ideas that are more likely to be absorbed with their existing knowledge during the activities (Van de Walle et al., 2009). This study also clarified what flipped classroom enhance students to develop competent problem solver because students were reflected about mathematics concepts, strategies, cognitions about problem solving, and metacognitive knowledge (awareness and control of their processing problem solving).

References

Amadeo, M. (2018). Textbooks revealing the development of a concept—the case of the number line in the analytic geometry (1708–1829). ZDM, 50:907–920

Atiyah, M. (2001). Mathematics in the 20th century. American Mathematical Monthly, 108(7), 654–666.

Bergmann, J., & Sams, A. (2012). Flip your classroom: Reach every student in every class every day. Washington, DC: Internal Society for Technology in Education.

Bishop, J. L., & Verleger, M. A. (2013). The flipped classroom: A survey of the research. In ASEE national conference proceedings, Atlanta, GA (Vol. 30, No. 9, pp. 1–18). Retrieved March 16, 2016, from https ://www.asee.org/publi c/confe rence s/20/paper s/6219/view.

Charles, R., Lester, F. and O'Daffer, P. (1987) How to Evaluate Progress in Problem Solving. Reston, VA: National Council of Teachers of Mathematics.

Cheng, L., Ritzhaupt, A. D., Antonenko, P. (2019). Effects of the flipped classroom instructional strategy on students' learning outcomes: a meta-analysis. Education Tech Research Dev, 67:793–824

Ebal Jr, C.D. Luga, M.J.F., Flores, M.R.O., Zabala, D.J.P. Buan, A.T., and Yuenyong, C. (2019). Linear Equations in Two Variables STEM Education Learning Activities: Developing the Household Power Consumption Calculator App. Journal of Physics: Conference Series, 1340 (1), 012048

Jackaria, P.M., Buan, A., Yuenyong, C. (2019). Students' Performance in Context–Based Lessons in Mathematics Classroom. Journal of Physics: Conference Series, 1340 (1), 012047

Jones, K. and Fujita, T. (2013) Interpretations of National Curricula: the case of geometry in textbooks from England and Japan. ZDM Mathematics Education, 45:671–683

Mariotti, M., and Balacheff, N. (2008). Introduction to the special issue on didactical and epistemological perspectives on mathematical proof. ZDM International Journal on Mathematics Education, 40(3), 341–344.

Mayer, R. E., and Wittrock, R. C. (2006). Problem solving. In P. A. Alexander & P. H. Winne (Eds.), Handbook of educational psychology (2nd ed.), (pp. 287-304). Mahwah, NJ: Erlbaum.

National Council of Teachers of Mathematics.(1991). Curriculum and evaluation standards for school mathematics. Geometry from multiple perspectives: Addenda Series, Grades 9–12. Reston, Virginia: National Council of Teachers of Mathematics.

Schoenfeld, A. H. (1985). Mathematical problem solving. London: Academic Press.

Setthaphongsakorn, S., and Yuenyong, C. (2019). Developing Ellipse Properties Learning Unit with Computer Assisted Learning through GeoGebra software for Thai High School Students. Journal of Physics: Conference Series, 1340 (1), 012051

Sinclair, N. (2008). The history of the geometry curriculum in the United States. Charlotte: Information Age Publishing.

Suanse, K and Yuenyong C. (2021). Development of the analytic geometry flipped classroom teaching model through Google Classroom. Journal of Physics: Conference Series, 1835 (1), 012077

Usiskin, Z. (1987). Resolving the continuing dilemmas in school geometry. In M. M. Lindquist & A. P. Shulte (Eds.), Learning and teaching geometry, K-12 (pp. 17–31). Reston: NCTM.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2009). Elementary and middle school mathematics:Teaching developmentally (7th ed.). Boston, MA: Allyn & Bacon/Merill.

Villani, V., et al. (1994) Perspectives on the teaching of geometry for the 21st century (discussion document for an ICMI Study). L'Enseignement Mathematique, 40(3–4), 345–357.

Woranetsudathip, N, Yuenyong, C, and Nguyen, TT (2021). The innovative lesson study for enhancing students' mathematical ideas about addition and subtraction through open approach. Journal of Physics: Conference Series 1835 (1), 012061