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The Innovative Lesson Study for Enhancing Grade 2 Students' Multiplication Conception through Open Approach

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Abstract: The paper aimed to clarify the innovative lesson study learning activities for enhancing Grade 2 students' multiplication conception. The paper provided the lesson study environment which allowed the primary school teacher to develop the lesson plan about multiplication conception through open approach. Regarding on the open approach, the cycle of lesson study focused how to provide students' openly constructing their meaning of multiplication based on everyday life situation. The lesson plans about multiplication concept were developed based on the situations of calculating amount of things in groups that each group has same number of thing. The paper will highlight these situations in order to visualize how they allow students to construct their concepts of multiplication and to generate multiplication symbols. The paper has implications for developing constructivist mathematics learning for Grade 2 students.

Keywords: multiplication, representation, connection, lesson study, open approach

1.Introduction

Learner-centered education arose from the constructivist learning paradigm. Thai educational reform is based on the constructivist philosophy. The learner-centered approach refers to learning methods that aim to grow people and enrich their lives. Learners should be provided with learning experiences that allow them to reach their full potential while also catering to their aptitude, interests, and requirements. Individual characteristics should be considered while organizing learning activities. They should allow learners to interact with people, nature, and technology that are relevant to their learning environment in everyday life (Tupsai et.al., 2015; Yuenyong and Thathong, 2015).

Regarding the philosophy of constructivism, mathematical understanding has often been described as involving both procedural and conceptual knowledge (Barmby et al., 2009; Rittle-Johnson, Schneider, & Star, 2015; Star, 2005). Procedural knowledge is characterized as step-by-step knowledge, such as how to do an algorithm calculation. (Maciejewski & Star, 2016; Rittle-Johnson et al., 2015). Conceptual knowledge is frequently stated as being linked to other "units of knowledge" (Hiebert & Carpenter, 1992) or as a "connected web of knowledge" (Hiebert & Lefevre, 1986). According to the literature, well-connected conceptual and procedural knowledge is defined as an indication of deeper comprehension in numerous frameworks for students' understanding in mathematics. (Baroody et al., 2007; Star, 2005). Indeed, connectivity is indicated as being essential to developing a deep and powerful knowledge. (Barmby et al., 2009; Baroody et al., 2007; Gray & Tall, 1994; Richland, Stigler, & Holyoak, 2012).

This suggests that in order to explore multiplication understandings, both procedural and conceptual knowledge, as well as the links between them, must be studied. Procedural knowledge can be observed in procedures, while conceptual knowledge and connections must be externally represented in order to be observed. A multiplication model can be represented either verbally as a word problem or visually as a drawing. Representations in one mode, such as visual, can be subdivided into diagrams, concrete materials, or drawings. Concrete things, such as manipulatives arranged in rows and columns or sketched drawings of a chocolate bar, are instances of rectangular array representations (Barmby et al., 2009). External representations can be used as thinking tools for the abstract mathematics they represent (Pape & Tchoshanov, 2001; Selling, 2016).

Understanding can be viewed as connections between representations of different types of knowledge, and the nature of connections proposed to be reasoning (Barmby et al., 2009). Reasoning is a significant research subject in mathematics education and is defined in a variety of ways, such as making generalizations and constructing arguments for whether generalizations are true or untrue (Stylianides, Stylianides, & Shilling-Traina, 2013; Suanse and Yuenyong, 2023)

Representation and connection provide ways of making sense of multiplication. Larsson (2016, p.30) showed that "the small cubes of the cuboid represent three-way connections. An example of a such a connection is to calculate $16 \cdot 25$ by use of the distributive property, as $10 \cdot 25 + 6 \cdot 25$, and connecting it to the model of equal groups by explaining that one can first calculate 10 of the groups and then the remaining 6 groups, irrespective whether knowledge of distributivity is implicit or explicit. Larsson, Pettersson, and Andrews (2017) discovered that the method teachers taught multiplication as repetitive addition was problematic, particularly when dealing with multi-digit and decimal multiplication. According to Chin and Jiew (2019), participants were requested to generate mathematical expressions based on real-life difficulties, i.e. from distinct real-life circumstances to symbolic. We believe it would be beneficial to investigate how participants translate their mathematical thoughts in real life into mathematical symbols.

The development of multiplicative thinking is described as a learning trajectory in four central phases: (1) direct counting, (2) rhythmic or skip counting, (3) additive thinking (possibly by saying the count-by sequence), and (4) multiplicative thinking (Anghileri 1989; Battista 1999; Downton and Sullivan 2017; Larsson 2016; Mulligan and Watson 1998; Ruwisch 1998; Siemon et al. 2005; Simon and Blume 1994; Sherin and Fuson 2005; Steffe 1992; Sullivan et al. 2001; Thompson and Saldanha 2003). In the beginning, repeated addition is thought to be more sophisticated than counting all or counting by multiples; nevertheless, equating multiplication with repeated addition is restrictive because this style of thinking is no longer possible beyond natural numbers. In contrast to additive thinking, multiplicative thinking requires the ability to coordinate bundled units on a higher abstract level and requires the recognition of the different meanings of the multiplier and the multiplicand. (Clark and Kamii 1996; Downton and Sullivan 2017; Larsson 2016; Singh 2000; Steffe 1992). This ability is often called 'unitizing' (Lamon 1994) or 'dealing with composite units' (Steffe 1992). However, many children struggle with the transition from additive to multiplicative thinking (Ehlert et al. 2013). Götze and Baiker (2021) argued that the study about how multiplicative thinking as unitizing should be supported in young children. They proposed that more study be conducted to provide information on how to construct multiplicative meaning-making processes and how youngsters can learn to think multiplicatively. Such meaning-making processes can be demonstrably assisted by linking forward and backward distinct mathematical representations (concrete, graphical, symbolic, and verbal) and language registers (everyday, academic, and technical) with a focus on verbalizing multiplicative structures.

Multiplication conception should be provided in the real-world problems. The multiplication conception problem should enhance students to learn mathematics related to their context and to develop their divergent thinking on problem solving**.** According to the literature, the mathematics problem should not be presented as a closed problem. Instead, open-ended issues should be used to challenge and encourage students to use divergent thinking and reasoning to develop their own concrete and informal problem-solving solutions. The open-ended tasks will introduce students to new mathematical situations, allowing them to happily and actively learn. Another aspect of providing a learning environment for constructing mathematical knowledge is the use of open-ended puzzles (Gravemeijer & Doorman, 1999; Pehkonen, 1995; Woranetsudathip & Yuenyong, 2015). In Thailand, Maitree Inprasitha drove open-ended problems in Thailand as an open approach, which was first adopted in Thailand mathematics classrooms in 2002. In addition, he proposed that the lesson study be used to build mathematics learning activities based on an open approach (Inprasitha, 2010; Kim et.al., 2019; Phaikhumnam & Yuenyong, 2018; Woranetsudathip et.al., 2021; Woranetsudathip & Yuenyong, 2015)

The lesson study helped teachers find effective approaches to teach mathematical concepts using an open approach (Woranetsudathip, 2021). According to the literature, the most effective area to improve teaching is in the context of a classroom lesson. The lesson study may enable us to develop and deliver mathematics lessons on multiplication using an open method (Stigler & Hiebert, 1999). Because teachers must form small groups (4-6 teachers) of teachers who teach children at the same level and/or material, the lesson study is a collaborative design of a research lesson. Then they start working collaboratively to design the lesson. After the lesson has been designed, a teacher from the group will teach it. The rest of the group will observe and collect data on the lesson process. Data collecting may concentrate on students' learning for the specific topic presented, as well as a range of students' learning concerns. Then, as individual reflections, each group member will provide his or her interpretation of classroom data collected to the group. Based on these reflections, the group must review and update the lesson for the following lesson. The new lesson plan will subsequently be taught to another set of students by another member of the group. Furthermore, the group may continue to observe classrooms and analyse data in order to provide feedback for future development (Matoba, 2005)**.** Many educators proposed the cycle of lesson study to suggest teachers adopting a culture of lesson study. One of those is about the cycle of plan, do, and see. Inprasitha (2010) proposed that lesson learning in Thai schools be adopted in three easy steps. These included collectively creating the research lesson (Plan), collaboratively witnessing a group member give the research lesson (Do), and collaboratively conducting a post-discussion or reflection on teaching practice (See).

According to the research and experiences on lesson study, it would be difficult for mathematics teachers to construct learning activities for young children (Grade 2 students) through open-ended mathematics problems. The lesson research may assist our group in determining effective methods of open approach mathematics teaching about multiplication for Grade 2 students.

2. Methodology

Methodology was concerned with the interpretive paradigm. The practice of lesson study helped to acquire an understanding of new learning activities. Participant observation, reflection of lesson study teachers, and generating lesson plan document were among the interpretive strategies used.

Method of inquiry

Based on Lesson Study, the initiative's new lesson study learning activities on mathematical multiplication were developed. The process includes 76 Grade 2 Thai children learning about mathematical theories on multiplication through an open approach in a classroom setting.

Teachers addressed ways to improve students' mathematical thoughts on multiplication using an open method based on lesson study. Teachers began to bring up real-world situations that were relevant to their pupils' context of multiplication. Teachers chose some real-world situations that could be related to Thailand's mathematics curriculum. One of the teachers decided to provide the lesson based on the lesson plan created in collaboration with his or her colleagues. The rest of the member group teachers served as active observers, taking notes on what happened in the classroom. The instructors then reconvened as a group to analyze, critique, and evaluate the lesson plan in order to assess the appropriateness of the teacher's performance, resources used, and challenges involved in boosting students' understanding of mathematical notions on multiplication. Finally, teachers discussed any changes to the lesson plan that were necessary based on their observations and reflections. The creative learning activities will subsequently be classified based on the lesson study.

3. Findings

Through the process of lesson study, the innovative lesson study learning activities of multiplication was developed into four lesson plans of open approach multiplication. The open-ended problems were provided to enhance student to develop representation and connection in multiplication. These included 1) making sense of multiplication via developing the new unit for counting, 2) Arranging objects into groups with equal numbers in each group and connecting on writing sentences with multiplication symbols, and 3) finding the number of items that have the same unit and writing a multiplication sentence.

3.1 Making sense of multiplication via developing the new unit for counting

Conception of multiplication could be constructed when students are counting things in various unit of counting. For example, we can count by one e.g. 1, 2, 3,… Some units, for example, we can count by two e.g. 2, 4, 6, 8, .. for unit of two eggs in boxes. And we can use various units to count by any. The lesson plan provided a party situation that there were many foods on the plate including apples, oranges, cake, donuts, strawberries and bananas as shown in figure 1. However, the number of apples in each dish is not the same. Unlike, other fruits and other desserts were provided in the same amount for every plate. This makes it a problematic situation for students. And it will be able to stimulate students to think about developing units of counting.

Figure 1: Plates of fruits and desserts for problem of developing units of counting

This lesson aims to connect students to understanding how to write symbolic sentences and the meaning of multiplication when the number of objects placed on the plate is the same. Students would be enhanced to make sense that the total number of objects can be represented by the number of plates and the number of items in each plate Including when the number of items placed on the plate is the same. Students would be enhanced to make sense that the total number of objects can be represented by the number of plates and the number of items on each plate. Then, they could develop the unit of counting regarding on things on a plate. In order to enhance students to develop unit of counting, teachers may provide the scaffolding questions. The following dialogue is examples of scaffolding for representation and connection on concept about multiplication.

Teacher: "What do students notice in the picture on the board?"
Student: In the picture there is a child, an apple, an orange, and a cake.
Teacher: "Observe the number of each item on the plate. How can students explain it?
Student: The number of oranges and cakes on each plate is the same. But the number of apples in each dish is different.
Teacher: "How many apples are there?"
Students: 9 apples
Teacher: "How do students find the total number of apples?"
Student: Count in increments of 1 according to the picture, 4 + 3 + 2.
Students work together to conclude that they used the counting or addition method to find the total number of apples.
………………………
Teacher: From finding the number of items placed on the table. There is the same number of dishes on each plate. How can we explain the total number of cakes?
Student: "Each plate has 2 cakes and there are 8 plates, so there are 16 cakes in total."
Teacher: How can we explain the total number of oranges?
Student: "Each plate has 4 oranges and there are 6 plates, so there are 24 oranges in total."
………………………
Teacher: You may learn that why we could easier count cakes and oranges than count apples.
Student: If the number of each item is the same on every plate, it will be easier to find the number of items.
Teacher: Can we do the same on counting apples? How can we do?
Student: We can move one apple from the plate that consists of 4 apples to a plate of two apples. Then, the number of apples on each plate is the same. (See picture 2)
………………………
Teacher: Regarding on counting the apples, can you explain the ideas of counting cakes, oranges, strawberry.
Student: "Each plate has 2 cakes and there are 8 plates, so there are 16 cakes in total."
"There are 4 oranges on each plate and there are 6 plates, so there are 24 oranges in total."
" Each plate had 7 strawberries, and there were 4 plates, so there were all strawberries.
28 results"

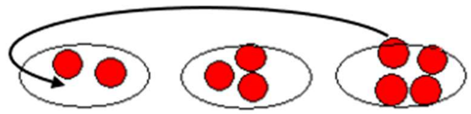

Figure 2: making the number of apples on each plate is the same.

3.2 Arranging objects into groups with equal numbers in each group and connecting on writing sentences with multiplication symbols

Students learned to represent and connect multiplication concepts through counting total numbers of things by arranging objects into groups with equal numbers in each group and writing sentences with multiplication symbols. The learning activity consists of 1) show how to find the number of items that have the same unit, 2) write sentences with multiplication symbols, 3) show how to arrange objects into groups with the same number in each group, and 4) tell the number in multiples. In order to enhance students to arrange object into groups and writing sentences with, teachers may provide the scaffolding questions. The following dialogue is examples of scaffolding for representation and connection on concept about multiplication.

Remind students about counting all objects that each group has the same number of items. Ask students look at figure 3 then teacher asks the students: Each plate has 3 donuts, and there are 5 plates total, so there are15 donuts in total.

Figure 3: same number of donuts in each plate

Ask students to excise the represent and connect multiplication concept through the following dialogue:

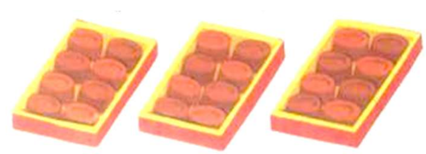

Figure 4: same number of chocolate in each boxes

Teacher: Students look at picture 4 and have each group write a message to indicate the number of objects.

Student: There are 8 pieces in each box, and there are 3 boxes, so there are 24 chocolates in total.

Figure 5: same number of fishes in each pack

Teacher: Students look at figure 5 and have each group write a message to indicate the number of objects.

Student: There are 2 fish in each pack, and there are 6 packs, so there are 12 fish in total.

Figure 6: same number of jellies in each bag

Teacher: Students look at figure 6 and have each group write a message to indicate the number of objects.

Student: There are 6 jellies in each bag, and there are 5 bags, so there are 30 jellies in total.

Figure 7: same number of pears in each plate

Teacher: Students look at figure 7 and have each group write a message to indicate the number of objects.

Student: There were 9 pears on each plate, and there were 2 plates, so there were 18 pears in total.

Ask students how to find the number of all objects to make connections with writing them in symbolic sentences. The following dialogue is examples of scaffolding for making connections with writing them in symbolic sentences about multiplication.
How many cakes are there? Together, they can be linked together to write a multiplication symbol sentence as follows.

See Figure 8. Each box has 2 items, and there are 5 boxes, so there are 10 items in total. Represent it with the symbolic sentence 2x5 = 10 and reads, "2 multiplied by 5 equals 10"
This type of calculation is called "multiplication."

Figure 8: same number of cakes in each box

3.3 Find the number of items that have the same unit and write a multiplication sentence.

The lesson plan of finding the number of items that have the same unit and writing a multiplication sentence was provided in the same steps of the second lesson plan however, this lesson plan was more focused on writing multiplication symbol and connection on counting to multiplication. The learning activity, therefore, consists of 1) show how to find the number of items that have the same unit, 2) write sentences with multiplication symbols, 3) show how to arrange objects into groups with the same number in each group, and 4) tell the number in multiples. The learning activity was provided to challenge students to apply multiplication concepts to explain everyday life situations. The following dialogue is examples of scaffolding students to represent and connect conception about multiplication in everyday life experiences.

Teacher: "What do you notice in the picture?"

Student: apple, fish, cake, fish, etc.

Teacher: "What are the characteristics of the objects or animals in the picture?"

Student: Some things are in groups. Some things don't belong in groups, things or animals that are in groups, some groups have the same number of members, such as fish, and some groups have an unequal number of members, such as fish, etc.

Students observed the figure 9 of the Let's Go to the Zoo activity that was given. Show how to find the number of things or animals that each group has the same number of group members by writing multiplication symbol sentences.

Students wrote the multiplication symbol sentences as following:

number of apples is $8 \times 2 = 16$
Number of dogs is $4 \times 2 = 8$
number of balloons is $2 \times 4 = 8$
number of horses is $6 \times 3 = 18$
number of birds is $5 \times 4 = 20$
number of cakes is $7 \times 4 = 28$
number of oranges is $9 \times 3 = 27$

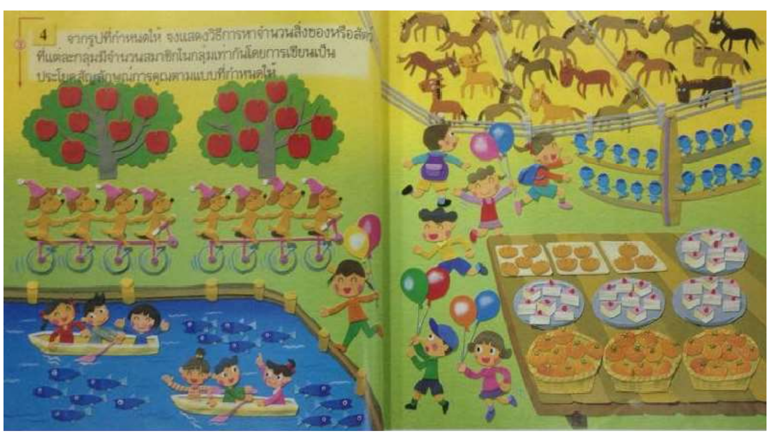

Figure 9: Let's Go to the Zoo (Inprasitha, 2011)

4. Conclusion

The study's most notable finding is the uncertain significance of equal groups and repeated addition in kids' understandings of multiplication. In this study, students understood multiplication as a technique of repeated addition, a model of equal groups, or a combination of the two. I don't think of repeated addition as a model for multiplication; rather, I think of it as a mathematical technique. Equal groupings and repetitive addition could be expected to play a significant influence. The multiplier effect, for example, is generally known to be based on a view of multiplication as matching multiplicative expressions (e.g., De Corte & Verschaffel, 1996). The robustness of the students' asymmetrical viewpoint, on the other hand, comprises details about advantages and disadvantages that are not reported for students in traditional education at the time when multi-digit and decimal multiplication is introduced. Different ideas can explain the persistence of equal groupings and recurrent addition. According to the intuitive model theory, repeated addition is deeply rooted and resistant to change for two reasons: it is the first multiplication method taught and "corresponds to features of human mental behavior that are primary, natural, and basic" (Fischbein et al., 1985, p. 15). According to the intuitive model theory, repeated addition will influence reasoning long after more generalised models and calculations have been added into the students' repertory. The long-term influence of initial training is widely accepted, however there are differing perspectives on the roots of multiplicative reasoning. For example, it has been proposed that the intuitive and informal concept of multiplication that children have prior to instruction is embedded in a one-to-many correspondence (Nunes & Bryant, 2010) or as splitting (Confrey & Smith, 1995), rather than as repeated addition, which distinguishes multiplication from addition conceptually.

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