

Development of Lesson Guide on Triangle Congruence as Teacher-support on Teaching Through Problem Solving

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Received: 15 Jul 2025

Revised: 22 Feb 2026

Accepted: 23 Feb 2026

Abstract. In mathematics education, recent studies have highlighted teaching through problem-solving (TTP) as an area of focus, with collegial support and guidance from experienced TTP practitioners being identified as key facilitators. The absence of support and appropriate materials for maintaining teaching techniques and problem-solving throughout the school year, as revealed in initial interviews with math teachers, led to the study's focus on developing a lesson guide for Grade 8 Triangle Congruence, identified as the least-learned competency in one public school. The participants in this study were chosen through convenience sampling. The TTP lesson guides are developed through the ADDIE Instructional Model. Results of the validation of the three lesson guides from two TTP practitioners showed that all were valid, with an overall mean of 3.56, 3.91, and 4.09 for each lesson guide. With revisions from their suggestions, it proceeded to the implementation phase. Two themes were generated from the Focus Group Discussion (FGD), namely (1) Supporting Professional Development of Teachers in Adopting the TTP Approach and (2) Fostering Student Engagement and Active Learning in Mathematics. Furthermore, the FGD showed that the developed lesson guide is acceptable as substantial support for teachers in sustaining TTP in the school.

Keywords: lesson guide, teaching through problem-solving, teacher-support

1. Introduction

Problem-solving is a central component of mathematics because it allows students to have an in-depth understanding of the mathematical concepts. It is one of the twin goals of the K to 12 Mathematics Curriculum Framework in the Philippines. According to Lester (2013), there are three approaches to problem-solving: teaching for, about, and through problem-solving. But recent studies have been leaning towards teaching through problem-solving (TTP) (Bailey, 2022). Unlike traditional approaches that treat problem-solving as an end-of-the-chapter application, TTP positions problem-solving as the primary vehicle for introducing and developing mathematical concepts.

George Polya's heuristic method for problem solving, developed in the mid-20th century, offers a practical framework for incorporating it into teaching (Martínez-Jiménez & Guerreiro, 2023). In Japanese mathematics, Takahashi (2021) explains that TTP lessons

begin with a problem designed to elicit multiple solution strategies, which are then compared and consolidated during structured whole-class discussion. This structure implies that in order to effectively implement TTP, teachers must carefully design cognitively demanding tasks that anticipate and draw out multiple student solution strategies. It also emphasizes the importance of structured whole-class discussion in integrating these strategies into coherent mathematical understanding, transforming the teacher's role from information provider to facilitator of conceptual development. Research shows that engaging students in problem exploration before instruction supports deeper mathematical understanding because it encourages students to reason and make sense of concepts rather than focusing on procedures alone (DeCaro & Rittle-Johnson, 2012) and conceptual scaffolding during problem solving further enhances students' persistence and meaning-making in mathematics (DiNapoli & Miller, 2022).

The success of TTP in Japan has prompted other countries to adopt and explore it through professional development training such as lesson study (LS), which is teacher-led research to improve pedagogical knowledge (Takahashi et al., 2013). By engaging in lesson study, teachers learn to implement teaching through problem solving (TTP), which supports the development of students' problem-solving skills through structured lesson design and classroom discussions (Roorda, de Vries, & Smale-Jacobse, 2024). A notable factor that affects its successful implementation is the presence of a fellow participating colleague, which is more pivotal in learning TTP than simply attending workshops (Bailey, 2022). This is especially true if it were a TTP-practicing teacher (Takahashi, 2016). However, the absence of this assistance affects novice teachers' ability to develop the necessary skills for the TTP approach.

Furthermore, TTP is also promoted in the Philippines within the capacity-building training program called Project Science Teacher Academy for the Region and the Collaborative Lesson Research and Development Project. The TTP approach holds a promising result based on the Filipino teachers' experiences (DOST-SEI & MATHTED, 2022; Ronda, 2013; Sionicio & Barbacena, 2021). However, it struggles to lay a solid foundation for Filipino teachers to persist in the use of TTP in their teaching practices. Ronda (2013) posits that teachers don't see the need for changes and are satisfied with their students' level of achievement; that's why they always go back to their old approach. King (2019) believes that this is because teachers don't have the core belief that solving genuine problems is the best way to learn mathematics. This mirrors the limiting factors laid out by Project STAR participants: lack of support and monitoring, willingness and commitment, and the intimidating effect of TTP. Thus, there is a need to figure out how to support teachers in the enhancement of their own teaching (Ronda, 2013).

In response to this issue, Japan and the US provided their novice mathematics teachers with textbooks to operationally guide them throughout the TTP process. The US LS highlighted the importance of having external support and suggested having an outside expert for the lesson planning (Takahashi et al., 2013). Drawing from these practices, a research-based lesson guide—modeled after a formal lesson plan and validated by TTP practitioners—was developed to support Filipino mathematics teachers seeking to implement a problem-solving approach in their lessons to enhance student performance.

The selection of triangle congruence as the focus of the lesson guide is both pedagogically and mathematically grounded. Triangle congruence is an important context for developing students' reasoning and proof skills, both of which are emphasized in mathematics education (Wang et al., 2018). Triangle congruence in a TTP framework allows students to formulate hypotheses, test cases, build counterexamples, and justify conclusions. Rather than memorizing congruence criteria, students investigate which conditions are required to uniquely identify a triangle. Students, for example, can use exploration and reconstruction tasks to determine which conditions are sufficient for congruence (Tessema et al., 2025). They may investigate whether three sides (SSS), two

sides and an included angle (SAS), or two angles and a side (ASA) ensure congruence, as well as why the side-side-angle (SSA) condition results in ambiguity. Such investigations are consistent with the core principles of TTP by putting reasoning and justification at the center of instruction.

Although Tessema et al.'s (2025) study focused on plane geometry in general, its findings on developing pre-service teachers' problem-solving and mathematization skills support using structured, problem-focused approaches like TTP when teaching concepts such as triangles in geometry instruction. According to them, some pre-service teachers showed a noticeable gap in their understanding of the relationship between triangle sides and interior angles. Thus, this study aimed to develop and evaluate Teaching Through Problem Solving (TTP) lesson guides on triangle congruence as a form of teacher support. Specifically, the study sought to (1) develop lesson guides aligned with TTP principles, (2) evaluate their quality and effectiveness based on feedback from TTP experts and mathematics teachers, and (3) explore teachers' insights regarding the usefulness of the guides in supporting problem-solving-oriented instruction.

2. Methodology

This study adopts a development research design. According to Sugiyono (2012), “development research is basic research activity to get information on the user needs (need assessment) and development to produce and test the effectiveness of these products” (as cited in Abi Hamid et al., 2017). The goal of this study is to develop TTP lesson guides, validated by TTP practitioners, to support teachers in practicing the TTP approach. The lesson guide development was focused on the ADDIE model, which is an acronym that refers to its five phases: analyze, design, development, implementation, and evaluation.

Analyze Phase. An interview with teachers was conducted among eight mathematics teachers of one public high school in Iligan City—the implementing school. They were asked questions to evaluate the need for the development of lesson guides and the identification of topics in geometry in order that researchers can be guided during the design and development phase. Subsequently, the selection of willing participants to be the implementer and observer was initiated in this phase. The implementation of the lesson guides automatically takes place in one of the implementer's Grade 8 classes.

Design Phase. In this phase, the researchers identified the number of lesson guides to be developed on the addressed least-learned competency. In this way, objectives aligned with the K to 12 mathematics curriculum and components of the lesson guides will be defined.

Development Phase. The development phase contains (i) the drafting of the problems and their possible solutions, (ii) validators' evaluation of the first drafted TTP lesson guides, and (iii) revisions made based on the comments and suggestions of the validators.

Table 1. Five-Point Likert Scale

Rating	Scale	Interpretation
5	4.20 – 5.00	Highly Valid
4	3.40 – 4.19	Valid
3	2.60 – 3.39	Moderately
2	1.80 – 2.59	Valid
1	1.00 – 1.79	Fairly Valid
		Not Valid

* This interpretation is adopted from Consebido (2022)

A separate group of teachers with prior TTP experience was selected as validators who reviewed and evaluated the first draft of the TTP lesson guide on the basis of the rubric adopted from Bataluna et al. (2021). The rubric is composed of eight components: learning objectives, materials, introduction, problem, possible solutions, development of concepts,

assessment, and conclusion. The following table shows the mean scaling used to evaluate the lesson guide. Before moving to the implementation phase, suggestions and recommendations from validators were used for improvement and revision of the lesson guides.

Implementation Phase. The validated lesson guides were implemented by one teacher in one of her Grade 8 mathematics classes in the implementing school. Before and after the lesson implementation, students were given the Achievement Test to compare the mean scores. The test was adopted from the Mathematics 8 Learner's Material of DepEd; therefore, there is no need for a validity and reliability test. In all implementations of the three lesson guides, two teachers' observers were in attendance.

Evaluation Phase. Immediately following implementation, the evaluation phase was initiated. This was conducted through a Focus Group Discussion (FGD). In order to understand the TTP lesson guide and how teachers view it as appropriate teaching material, the FGD was held between the implementer, two observers, and four researchers.

3. Results and Discussion

The presentation of the results is for the following objectives: (1) development of the lesson guide in triangle congruence, (2) implementation of the developed lesson guide, and (3) evaluation and insights on the developed lesson guide as teacher support on TTP.

3.1 Analysis, Design, and Development of the TTP Lesson Guides

In the analysis phase, it was found out that six of the eight mathematics teachers stated being familiar with TTP; one demonstrated an occasional use, while the other two teachers stated that they were not. Seeing the potential of the TTP approach, one teacher raised a concern regarding the ability of mathematics teachers to formulate a problem. The mathematics teachers also showed reluctance because of the additional work. However, when asked if a ready-made lesson guide would support them in TTP, they showed a positive response. On the other hand, according to three Grade 8 mathematics teachers among the eight, the two least-learned topics for geometry are namely Triangle Congruence and Triangle Inequality—a 3rd quarter and 4th quarter topic, respectively. However, triangle congruence was chosen as the focus of this study because it is a core geometry topic that plays a key role in developing students' reasoning and is emphasized differently across mathematics curricula (Luo, Zhou, & Liu, 2024). According to Masinading & Gaylo (2022), "the majority of students have difficulties understanding basic triangle congruence concepts, including stating and illustrating SAS, ASA, and SSS congruence postulates." This is supported by the study conducted by Laña (2023), which shows that illustrating SAS, ASA, and SSS congruence postulates ranks first as the least-learned competency in Mathematics 8 in the 3rd quarter.

In the design phase, the topic "Triangle Congruence" was divided into three subtopics for each lesson guide taught through problem solving: Correspondence and Congruence, Triangle Congruence Postulates, and AAS Triangle Congruence Theorem. To ensure alignment with the national standard set by the K-12 Mathematics Curriculum Guide, the researcher identified and mapped the standards. The creation of the lesson guides incorporated the principles of the Teaching Through Problem-Solving (TTP) approach. With a focus on the specific objectives identified in the analysis phase, the researchers structured engaging and coherent lessons that aligned with DepEd's format. Throughout the designing phase, the researcher remained mindful of promoting student engagement and active participation, as well as the integration of problem-solving skills. By combining the TTP approach with the DepEd's format, the researchers were able to create well-structured lesson guides that effectively addressed the learning objectives and set the stage for the subsequent development and implementation phases.

The details for the development phase were as follows:

(i) *Drafting of the Problem*

The 1st problem was taken from a Japanese textbook discussed in the study of Jones & Fujita (2013), “Characterizing Triangle Congruency in Lower Secondary School: The Case of Japan,” which is an introduction to congruent figures. The researchers chose this problem, and it was contextualized because of its potential for understanding the idea of correspondence and congruence through physical and mental overlapping of the triangles

The goal of the 2nd problem is to derive the Triangle Congruence Postulates from the students. While students draw the congruent triangles, they will discover that not all six measurements of the triangles are needed but only a minimum of three (side-angle-side, angle-side-angle, and side-side-side). This problem is one of the *Spotlight Lessons* compiled by Takahashi (2021) in his book “Teaching Mathematics through Problem-Solving: A Pedagogical Approach from Japan,” where there’s a section of TTP lessons that teachers can use.

Using the same idea in lesson guide 2, the researchers formulated the 3rd problem to investigate the AAS Congruence Theorem. The researchers decided to separate it from the Triangle Congruence to set a difference between a postulate and a theorem. Moreover, the students can associate AAS congruence to ASA congruence.

(ii) *Evaluation of the lesson guides*

Table 2 summarizes the mean ratings of the two validators on the three TTP lesson guides. A mean of 3.40 to 4.19, which is interpreted as *Valid*, was given for each lesson guide. It means it can proceed to the implementation stage without re-evaluation. However, some parts of the lesson guides have low ratings, specifically the introduction of each lesson guide with 2.5–3.0 ratings and the possible solutions of lesson guide 1 with a rating of 2.5. This indicates that there are areas that need improvements before it proceeds to the implementation phase.

Table 2. Summary of Validators’ Ratings of the TTP lesson guides

Category	Description	MEAN		
		LP 1	LP 2	LP 3
<i>Learning Objectives</i>	Lesson objectives are stated in clear, measurable, and observable terms; Are aligned to the K-12 standards.	4.5	4.5	4.5
<i>Materials</i>	Detailed list of materials is provided for both teacher and students; All handouts, both teachers created and those from other resources, are referenced in the procedures and attached to the lesson guide.	3.5	3.5	4
<i>Introduction</i>	Introduces the lesson by sharing purpose, relevance; with clear overview of the student activity.	2.5	3	3
<i>Problem</i>	1) The problem has important, useful mathematics embedded in it; 2) The problems can be approached by students in multiple ways using different solution strategies; 3) The problem contributes to the conceptual development of the students; 4) The problem requires higher-level thinking and problem-solving	3.75	3.88	4
<i>Possible Solutions</i>	Reflects possible solutions/means of solving the problem.	2.5	4	4.5
<i>Development of Concepts</i>	The development of concepts effectively relates to the anticipated solutions. The students are given more than adequate opportunity to ask questions and are encouraged to interact with their classmates. The teacher often asks higher order thinking questions.	3.5	4	4.5
<i>Assessment</i>	Formative and summative assessments are defined, showing a clear relationship to all objectives addressed in the lesson	4	4.5	4.5
<i>Closure</i>	Students review the lesson by summarizing and/or sharing what they learned, teacher revisits the purpose for the lesson	3.5	4	4
WEIGHTED MEAN		3.56	3.91	4.09
Interpretation		<i>Valid</i>	<i>Valid</i>	<i>Valid</i>

(iii) *Comments, Suggestions and Revisions of the Developed lesson guides*

The succeeding discussions are part of the validators' rating of the developed lesson guides. It contains comments and suggestions during the evaluation, which were the basis of the revision

Lesson Guide 1: Correspondence and Congruence. There were components of lesson guide 1 that were rated below the *Valid* scale. As such, the researchers made revisions mostly on the introduction (*activating prior knowledge*), the problem and possible solutions (*presenting and modelling*). Upon examination, the researchers found out that these parts of the lesson greatly affected the flow of the teaching-learning process.

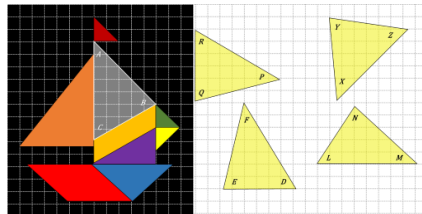
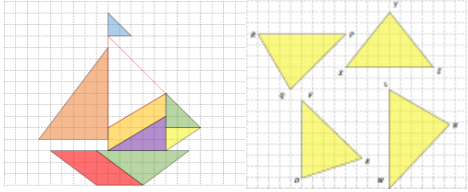
Table 3 summarized the comments and suggestions of the validators to improve lesson guide 1. In the introduction, students are given two additional activities before moving on to the main activity. First is a short review of the polygons that can be used in tangrams. Second is an introductory activity connected to the problem (finding the missing piece of the tangram) to address the comment of V_2 . As pointed out by V_1 the questions in the development of concepts are too general. It was suggested to be based on the aspects of the solutions and representations.

Table 3. Summary of Revisions for lesson guide 1: Correspondence and Congruence

Component	Before	Comments/Suggestion	Action for Revision
Introduction	An introduction relating to problem-solving activity.	The so-called establishing the purpose should inevitably touch on the previous knowledge for coherence and connection, not simply an "introduction". – V_1 Make a detailed information about your problem so that the students will fully understand or comprehend the problem. – V_2	Review of quadrilaterals and playing with the tangrams. Students should figure out that there is a missing triangle.
Possible Solution	The students are expected to overlap the triangles and measure the sides and angles.	The anticipated solutions are too limited. The reasoning is very limited. Very fixed to "right" answers. Can be improved by making the choice triangles be more intentional, or making the problem more open. – V_1	The illustrations are mapped onto the grid. The students are expected to use the area of the triangle as other emerging solutions.
Development of Concept	The guide questions are too general.	Ask questions to students from simple to difficult so that the students will be encouraged to participate in the activity. – V_1	There are guide questions for each possible solution and are sequenced from simple to difficult.

Table 4 exhibits the changes made in lesson guide 1 from the comments and suggestions of the validators. These suggestions emphasize the simplification of the lesson guide that is easily understood by the students with an increasing complexity that enables them to build their own ideas. In addition, the anticipated solutions from the students should be maximized to help teachers facilitate and build the concepts when they use the lesson guide.

Table 4. Sample Changes in lesson guide 1

Drafted Lesson Component	Revised Lesson Component
Introduction	
Establishing the Purpose of the Lesson Today we are going to learn how to identify congruent triangles from the given triangle ABC and the other four triangles. With this, we will be able to name the corresponding parts of congruent triangles and define what congruent triangles are.	Review: <i>The teacher presents some figures and asks the students to name each figure shown.</i>  Establishing the Purpose of the Lesson <i>The teacher will have a group activity and will group the class into 8 with 5-6 members per group and provide cutouts for students to proceed with the short activity.</i>  Activity 1: <i>Direction: Use the given cutout shapes to form the figure shown below. Work with your groupmates and form it within 2 minutes.</i> <i>The teacher will roam around to see the progress of each group and to determine who finishes first in the given activity. The teacher will ask what figure you formed?</i> - Is there a missing shape to form a figure? - What shape is the missing?
Possible Solution	
Analysis  Solution 1: Measurement <i>Using the measurements of the missing triangle that Joseph needed in his tangram, students measure the triangle sides and angles using a ruler and protractor. The measure of the sides of ABC are $AC = 8$ cm, $AB = 7.2$ cm and $BC = 5.8$ cm. Meanwhile, the measure of the angles are $\angle A = 44^\circ$, $\angle B = 76^\circ$ and $\angle C = 60^\circ$. The triangle with the same measurement is PQR. Therefore, PQR will overlapped (congruent) with ABC.</i> Solution 2: Mapping One to the Other <i>Students may cut the triangles to directly see which triangles overlaps with ABC that Joseph needs to have.</i> - A corresponds to P of PQR. - B corresponds to Q, and C corresponds to R.	Analysis  Solution 1: Measurement <i>Using the measurements of the missing triangle that Joseph needs in his tangram, we measure the triangle sides and angles using a ruler and protractor. The measure of the sides of the missing triangle is: the first side measures 8 cm, the second side has 7.1 cm, and the third side has 5.8 cm. Meanwhile, the measurements of the angles are: the first angle is 44°, the second is 76°, and the last is 60°.</i> <i>The triangle with the same measurement is the first triangle. Therefore, the first triangle will be the missing triangle that Joseph needs to complete his tangram.</i> Solution 2: Mapping One to the Other <i>What we did was cut the triangles to directly see which triangle overlaps with the missing triangle that Joseph needs to have. We tried to overlap each triangle in the missing triangle that Joseph has in his tangram. With this, we conclude that the first triangle is the one we are looking for, as it is the only triangle that overlaps in the missing triangle figure in Joseph's tangram.</i> Solution 3: Finding the Area of the Triangles <i>Our group decided to measure first the base and height of the missing triangle that Joseph has in his tangram using a ruler and use its measurement to find the area of the missing triangle. Then, we identify the</i>

Similarly, $\angle A$ corresponds to $\angle P$ of PQR, $\angle B$ corresponds to $\angle Q$, and $\angle C$ corresponds to $\angle R$. For the sides, line segment AB corresponds to PQ of PQR, AC corresponds to PR, and BC corresponds to QR.	other areas of the four triangles in the choices given to determine which one has the same area as what Joseph needs to have in order to complete his tangram. Based on the results, we conclude that the 1st triangle and the 3rd triangle are what Joseph needs to complete his tangram.
<i>Development of Concept</i>	
(After explaining their answers, the teacher will present the answers to see if they got the same solution) - What did you notice with your answers with the other groups? - What did you consider in finding the triangle that overlaps with ABC? - What did you observe the sides and corners of the two triangles? (project: correspondence) - How we can write the corresponding angles and corresponding sides of the two triangles - We know that the corresponding angles and corresponding sides have the same measurement. We conclude to the PQR that overlaps with ABC? (project: congruence) - So how can we write that the two triangles are congruent? - To sum up, what makes the two triangles congruent? - Is it necessary to find the correspondence of the triangles when they are congruent?	(After explaining their answers, the teacher project the missing triangle and check if they got the same triangle) - Does your triangle overlap with Joseph's missing triangle? <i>Suggested Guide Questions:</i> <i>In solution 1</i> - What makes you conclude that the missing triangle is triangle 1? - How about the other triangle? The triangle 2 can fit the missing triangle. Why not? - How about the triangle 3? Triangle 3 can fit the missing triangle. Why not? - How about triangle 4? The triangle 4 can fit the missing triangle. Why not? - In your observation, when can we say that the triangle we found overlapped/ fit the missing triangle? - What do we call two triangles whose measurements are equal with each other? (Project: congruent triangles) - How about the two triangles having the same angles and but vary in size? Can we still conclude that they are congruent? Just like triangle 4 and triangle ABC. <i>In solution 2</i> - In the first solution if the two triangles have the same measurement of the sides and angles it will overlap each other. - In overlapping the two triangles what did you first consider to conclude that it fits together? (Project on the screen: Corresponding parts) - So if the two triangles overlap with each other, the two triangles will be called? - So we can conclude that triangles ABC and triangle 1? <i>In solution 3</i> - What triangle has the same area of Joseph's missing triangle? - Does this mean triangle 1 and triangle 3 are congruent to the missing triangle? From your statement, Can we conclude that if the two triangles have the same area are congruent?

Lesson Guide 2: Triangle Congruence Postulates. Table 5 shows the summary of the revisions made for lesson guide 2.

In this lesson, the main concern of validator 1 is that it is going in a deductive direction. Instructing the students to find the minimum requirement to create a congruent triangle is what makes the lesson deductive; thus, a revision was made for the task given in lesson 2. In the same component, the characters involved in the problem were reduced to two—Juan and Jose—to make it less confusing to students. The researchers realized that three characters can be associated with the three sides or corners of the triangles.

Table 5. Summary of Revisions for lesson guide 2: Triangle Congruence Postulate

Component	Before	Comments/Suggestion	Action for Revision
<i>Problem</i>	There are three characters to the problem.	“the idea that the three persons are either corners or lengths is confusing to me.” – V_I “Please change the names of the people involved by the names of our native people here the Philippines.” – V_I	Two characters, Jose and Juan, are involved in the problem.
<i>Development of Concept</i>	The questions are going in a deductive direction.	“This is not the goal of TTP. You have to be inductive.” – V_I	The question in the problem.

Lesson Guide 3: AAS Congruence Theorem. The suggested action for revision in lesson guide 3 was about the introduction and the problem (see Table 6).

Table 6. Summary of Revisions for lesson guide 3: AAS Congruence Theorem

Component	Before	Comments/Suggestion	Action for Revision
<i>Problem</i>	The problem involved different characters to the previous lesson but has similar activity.	“You can connect directly the problem as you have yesterday.” – V_I	The same characters are involved and the activity is just a continuation of the previous lesson.

Since the AAS congruence theorem touches on proving congruent triangles, V_I suggested connecting the problem to the problem in lesson guide 2. An additional suggestion from V_I was to change the approach of the teacher.

All the revisions made by the researcher in the development of TTP lesson guides were based on comments and suggestions from both validators after submitting the first draft. The developed lesson guides are ready for the implementation after their completion.

3.2 Implementation of the TTP lesson guides

This section (i) describes the implementation of the lesson guides and (ii) reports the results of the Grade 8 students based on the pre-test and post-test scores on the Achievement Test.

(i) Implementation of the TTP lesson guides

Before the implementation started, the Grade 8 students answered the pretest the day before. Each of the TTP lesson guides underwent implementation for one day.

(ii) Achievement Test Results.

Table 7 presents the statistical analysis between the pre-test and post-test results based on the information obtained before and after the implementation of the developed lesson guides.

Table 7. Statistical Results

Group	Paired Statistics					Paired Differences		
	N	Min Score	Max Score	Mean (M)	Standard Deviation (SD)	Mean Difference	t	p
Pretest	40	5	11	7.525	1.840	3.375	-9.083	< .001
Posttest	40	6	15	10.900	2.193			

The result of the pretest shows a mean score of 7.525 with a lowest score of 5 and a highest score of 11 out of 20 multiple-choice questions. This indicates that students have little background with triangle congruence at the beginning of the lesson. Meanwhile, the posttest result shows a mean score of 10.900 with a lowest score of 6 and highest score of 15 out of 20. Only 0.900 more than the 50% mark. This indicates the difficulties of the students in understanding and illustrating the Triangle Congruence and why it is one of the least competencies in Grade 8 Geometry (Masinading & Gaylo, 2022; Laña, 2023).

3.3 Evaluation

After the implementation of the developed lesson guides, the researchers conducted a focused group discussion (FGD) among the observers, implementers, and the researchers themselves. The result of the FGD was the basis for objective two of the study, specifically for the evaluation of the developed lesson guides from the observers and the implementer as a basis for its final revision, and the last objective, which asks about their insight into the developed lesson guides as teacher support for the TTP approach. The themes generated from the discussion addressed the impact of the developed TTP lesson guides on the students and the teachers, namely (1) Supporting Professional Development of Teachers in Adopting the TTP Approach and another emerging theme, which is (2) Fostering Student Engagement and Active Learning in Mathematics. The suggested revisions of the developed lesson guides based on the observations of the implementer and observers will also be discussed to further improve their quality and effectiveness.

Supporting Professional Development of Teachers in Adopting TTP Approach

The TTP lesson guides that have been developed support mathematics teachers in the TTP approach, which was agreed upon by the implementer and the two observers. As mentioned by one of the observers, it serves as a guide, particularly for teachers who are still new to the said approach and think it is a valuable tool in the field of teaching. Aside from that, the implementer found the TTP lesson guide helpful since one of its features contains the students' anticipated response along with guide questions that would help build the critical thinking skills of the students. However, there's a concern brought out by one of the observers, who states that TTP is done on a case-by-case basis. This is because of the contrasting characteristics of mathematics teachers, which influence their view of TTP. The implementer also pointed out that it is difficult to insist on this method with teachers who are accustomed to the traditional approach. During the discussion among the implementer and two observers, they emphasized the importance of supporting the professional development of the teachers. They provide two suggestions when asked what the best way is to sustain the practice of the TTP approach and to support teachers professionally, and that is monitoring and collaboration. The three teachers see the need for mandatory monitoring of TTP lessons inside the school, specifically the involvement of an expert as an evaluator of the lesson guides and as an observer during the implementation

Fostering Student Engagement and Active Learning in Mathematics

The discussion among two observers and the implementer revolves around the importance of problem-solving and critical thinking in TTP lessons. They found that these lessons enhance students' problem-solving and critical-thinking skills by enabling them to think and make real-world connections. This acknowledges the positive impact of problem-solving activities on cognitive development and the broader implications of teaching independent and analytical thinking skills, emphasizing their relevance and transferability to real-world scenarios. The significance of teaching students to think during the implementation of the developed lesson guides was also observed. This is evident in the implementer's observation, which underscores the depth of engagement and the level of cognitive involvement demonstrated by students during the learning process.

And the appreciation for hands-on learning experiences emphasizes the value of interactive, experiential learning approaches in facilitating student engagement and promoting active thinking. Additionally, the implementer noticed a significant change in the engagement of advanced learners that may not have been apparent through a traditional teaching method. This implies that traditional teaching may not adequately stimulate intellectually advanced students, leading to disengagement and boredom. However, the given problems elicited a positive response mostly from intellectual students, and little was demonstrated by the other students on the first day of implementation. The teachers made a notion that this method is still new to the students and needs adjustment.

4. Conclusion

The findings indicate that the development of Triangle Congruence lesson guides grounded in the Teaching Through Problem-Solving (TTP) approach can contribute to improvements in mathematics instruction. Employing the ADDIE Instructional Model demonstrates how a structured and standards-aligned design process can support the integration of TTP principles while maintaining instructional coherence and quality.

Validation results from TTP experts and mathematics teachers show that the lesson guides are appropriate for classroom implementation. Their use highlights the potential of TTP-based materials to support student engagement, active participation, and higher-order thinking through problem-solving activities. The inclusion of real-world and experiential tasks further underscores the importance of contextualized learning in promoting deeper conceptual understanding.

Feedback from teacher participants points to the role of continuous professional development in addressing challenges related to implementation and in sustaining innovative instructional practices. Collaborative mechanisms such as Learning Action Cell or Lesson Study sessions appear to be particularly valuable in fostering reflective practice, peer support, and shared responsibility for instructional improvement.

Taken together, these findings underscore the potential of wider adoption of the TTP approach, particularly when supported by well-designed instructional materials and ongoing teacher development. Such conditions may help create a more student-centered and intellectually engaging mathematics learning environment that better responds to both academic expectations and real-world learning demands.

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