A COLUMN GENERATION TECHNIQUE WITH MULTIPLE SUB-PROBLEMS FOR 2- DIMENSIONAL CUTTING STOCK PROBLEM

เทคนิคคอลัมน์เจเนเรชันหลายปัญหาย่อยสำหรับปัญหาการตัดแผ่นวัตถุในสองมิติ

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Abstract

In this paper, we discuss two-dimensional cutting stock problems (2DCSP) where the rectangular panels of difference sizes must be cut from standard multiple-size rolls. Moreover, the panels have to be obtained through two-stage guillotine cuts. The objective is to minimize the number of used rolls (or the area of waste). A new method based on column generation technique with multiple sub-problems is introduced to solve the problem. Various types and numbers of sub-problems are tested using real world data instances from electronic board industry. The computational results show the impact of solutions from difference sets and on average yield approximately 28 percent reduction of the waste area comparing with a basic method.

Keywords: branch-and-price, column generation, cutting stock problem, guillotine cut

บทคัดย่อ

ในบทความนี้ได้นำเสนอปัญหาการตัดแผ่นวัตถุในสองมิติเมื่อแผ่นวัตถุสี่เหลี่ยมเล็กขนาดต่างๆ ได้มาจากการตัดแผ่นวัตถุดิบที่มีขนาดสี่เหลี่ยมมาตรฐานที่มีหลากหลายขนาด นอกจากนี้การตัดแผ่นวัตถุดิบสี่เหลี่ยมมาตรฐานจะต้องตัดในลักษณะของการตัดแบบกิโยตีน 2 ขั้น โดยมีจุดมุ่งหมายคือ การใช้แผ่นวัตถุดิบสี่เหลี่ยมมาตรฐานจำนวนน้อยที่สุด หรือทำให้เหลือเศษจากการตัดน้อยที่สุด งานวิจัยนี้ได้นำเสนอวิธีใหม่ในการหาคำตอบซึ่งมีพื้นฐานมาจากเทคนิคคอลัมน์เจเนเรชันที่ประกอบไปด้วยหลายปัญหาย่อย โดยทำการทดสอบกับข้อมูลจริงจากอุตสาหกรรมผลิตแผงวงจรอิเล็กทรอนิกส์ ผลจากการทดลองพบว่า ชุดคำตอบเริ่มต้นที่แตกต่างกันมีผลกระทบกับคุณภาพของคำตอบ นอกจากนี้คำตอบที่ได้ส่งผลให้ลดปริมาณเศษจากการตัดโดยเฉลี่ย 28 เปอร์เซ็นต์ เมื่อเปรียบเทียบกับวิธีหาคำตอบพื้นฐาน

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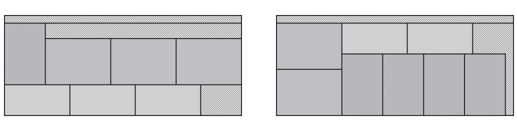
Introduction

The cutting stock problem deal with how to divide the large objects (sheets) into the small objects (panels) with minimum waste or trim loss. This problem is often found in can or wood production and metal industry. The structure of this problem is extended from the original bin packing problem where the demands of small objects are more than one. For the cutting pattern design, the panels must be placed on the sheet in a suitable position according with the ability of the cutting machine. In this study, we focus on 2-staged guillotine cutting pattern that is the pattern with the following properties:

- The cutting lines are parallel to the side of the sheet from one side to the other.
- The first stage cut is the cut on horizontal line and the second stage cut is the cut on vertical line in the sheet.
- We allow to split the sheet into two or three sub-sheets by guillotine cut before designing a cutting pattern.

In Figure 1, we give an example of 2-staged guillotine cuts on a rectangular sheet (a) and a non- guillotine cut (b).



(a) (b)

Figure 1 Guillotine cuts (a) and Non-guillotine cut (b)

In this study, we proposed a new method to solve 2-dimensional cutting stock problem using column generation technique with multiple sub-problems. We also investigated the solution quality when two sets of initial solution are different. The paper is organized as follows. First, we mention the studies about the 2DCSP. Second, the cutting stock problem is described by mathematical model in set-partitioning form with the column generation procedure. Third, the real-world data instances are tested using a proposed algorithm. The computational results from previous section are discussed and concluded in the last section.

Literature Reviews

The 2DCSP has been widely studied in the operational research and optimization field. There are several methods to handle the problem, such as formulating the problem in mathematical model, designing heuristic algorithms and applying the meta-heuristic.

Riehme, Scheithauer & Terno (1996) presented the cutting stock problem when the patterns are generated as 2-staged cutting pattern with difference sizes of sheet and a large range of the number of demands of the panel.

Furinia & Malaguti (2013) considered the 2DCSP with multiple stock sizes. Three Mixed-Integer Programming models are proposed in the literature. The first and the second models can be solved with a general-purpose MIP solver. The last model with an exponential number of variables is solved by column generation techniques and branch-and-bound to find the integer solution.

Lodi & Monaci (2003) presented 2 models for 2-staged 2-dimensional knapsack problems, which is the cutting stock problem with a unique rectangular sheet. They observed that there are multiple optimal solutions with the difference set of used cutting patterns. Therefore, they constructed the models based on this observation and tested the models by using then standard branch-and-bound method. The model of this problem can be applied to a sub-problem in 2DCSP using column generation.

Cintra et al. (2008a) investigated a dynamic algorithm based on a column generation heuristic to tackle the two-dimensional cutting stock and strip packing problems.

Furini et al. (2012) also proposed a column generation heuristic, which requires as its sub-problem the solution of a two-dimensional knapsack problem (2DKP). The 2DKP is solved in two phases by a dynamic algorithm. In the first phase, this problem is transformed to the one-dimensional knapsack problem with respect to the width of the sheet. The solutions of this phase are the set of potential strips (or shelves). The second phase due with using the strips from the first phase to build the pattern with respect to the width of sheet. Certainly, the solution of this algorithm does not guarantee the optimal solution but the computational experiment shows the effectiveness of the algorithm, which obtains very small optimality gaps.

Alvarez-Valdes, Parajon & Tamarit (2002) developed several heuristic methods to solve two-dimensional cutting stock problem based on column generation. By using dynamic programming, the sub-problem is solved to the attractive columns with reduced cost.

Another idea to design the cutting patterns was found by Yanasse & Morabito (2008b) who proposed the new ways to design cutting patterns by separating the original sheet into 2 sub-sheets (or 2-group) and 3 sub-sheets (or 3-group). The integer linear programming models for two-dimensional guillotine cutting patterns, including exact and non-exact cases, are proposed in this article and these models can be used to construct the sub-problem in the column generation technique.

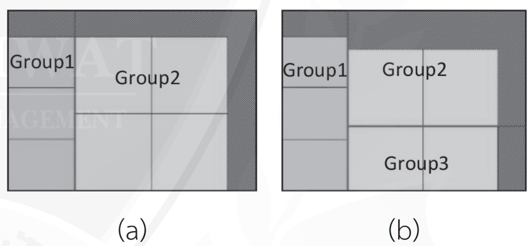


Figure 2 2-group pattern (a) and 3-group (b)

Normally column generation has only one sub-problem for generating attractive columns (patterns). For the 2DCSP problem, there are several models of the 2DKP with guillotine cut to design the patterns. Therefore, we proposed a new method and investigated the impact of sub-problems when A column generation technique has multiple sub-problems.

Column Generation

A column generation technique is used to solve a combinatorial problem with many decision variables. This technique is based on Danzig-Wolfe decomposition. Let $S_j (j = 1,....,n)$ be the family of all feasible cutting pattern of sheet *j*. The decision variable $x_p (p \in S_j)$ denotes the number of times when the cutting pattern *p* is used in the solution. We define the waste from cutting pattern *j* by A_p. Let C_p^i represent the number of panel $i (i = 1,...,m)$ in the cutting pattern *p* and d_i represent the requirement of panel *i*. 2DCSP can be modeled as follows.

$$Min \ \sum_{j=1}^{n} \sum_{p \in S_j} A_p x_p \quad (1)$$

Subject to:

$$\sum_{j=1}^{n} \sum_{p \in S_j} C_p^i x_p = d_i \ , \ i = 1, \dots, m \quad (2)$$

$$x_p \geq 0 \ and \ Integer \ , \ j = 1, \dots, n \ , p \in S_j \quad (3)$$

The above model is called *Master problem (MP)*. The objective function (1) is to minimize waste of used sheets. The constraints (2) ensure that the number of panel *i* must equal the number of requirements. Finally, the constraints (3) are the integrality constraints.

MP requires that all patterns must be specified. However, we cannot generate all feasible patterns for large-size instances. Therefore, a restricted master problem (RMP), which consists of a subset of patterns of MP, is used instead. To obtain the optimal solution, we generate a sub-problem (for each sheet *j* with length (*L*) and width (*W*)) using the dual solution from the current solution in RMP. The solution of the sub-problem either gives an improving column (pattern) to RMP or indicates that the current solution is optimal.

Initial Patterns

The set of partial feasible cutting patterns was generated by a simple algorithm with a short computing time. In the initial set of patterns, there must be the combination of numbers of used patterns to satisfy the demand for each panel. For improvement of final solution and investigation of initial patterns impact, the algorithm is tested using two sets of initial patterns. The first set, called *Int*1, is obtained from an algorithm which associates each panel type with a sheet type. In case all required panels of type *i* can be cut on only a single sheet, the algorithm selects the pattern on the sheet with the smallest waste area. On the other hand, if it cannot cut all required panels of type *i* on one sheet, the algorithm generates the pattern on the sheet with the maximum number of panels of type *i*. In addition, the algorithm also generates the patterns with the number of panels of type *i* starting from 1 to the maximum number of panels. Another initial patterns set, 2, is obtained by selecting a sheet that is large enough to contain a single panel of any types. The patterns are generated by putting one type of panel on this sheet with one panel per sheet. Therefore, the number of patterns in this initial set is equal to the number of panel types.

Sub–Problems Formulation

Three types of designing cutting patterns were used in this method as a sub-problem. The first sub-problem deals with a mathematical model of two-dimensional two-staged knapsack problems with guillotine cuts (2DKP) that was proposed by Lodi & Monaci (2003). The objective of the sub-problem is to find a pattern which can be cut a first stage on horizontal line and a second stage on vertical line with the maximum cost. Therefore, solving this problem gives a pattern with the most negative reduced cost, which can also be used to prove optimality. A mathematical model are shown which involves integer variables x_{ik} denoting the number of panels of type i (i = 1, …, m) in shelf k (k = 1, …, α_i) and q_k (k = 1, …, $\bar{n}$) denoting whether a shelf k is used (where n is the number of panels and $\alpha_i = \sum_{s \le i} d_s$).

Let be a dual solution from the current optimal solution in RMP associated with panel. The mathematical model the sub-problem for each sheet type is shown below:

$$Max\ \ \sum_{i=1}^{m} \pi_i^* \left(\sum_{k=1}^{\alpha_i} x_{ik} + \sum_{k=\alpha_{i-1}+1}^{\alpha_i} q_k\right) \qquad (4)$$

Subject to:

$$\sum_{k=1}^{\alpha_i} x_{ik} + \sum_{k=\alpha_{i-1}+1}^{\alpha_i} q_k \le ub_i \ \ , i = 1, \dots, m \qquad (5)$$

$$\sum_{i=\beta_k}^{m} \bar{w}_i x_{ik} \le \left(W - \bar{w}_{\beta_k}\right) q_k \ \ , k = 1, \dots, \bar{n} \qquad (6)$$

$$\sum_{k=1}^{\bar{n}} \bar{l}_{\beta_k} q_k \le L \qquad (7)$$

$$\sum_{s=k}^{\alpha_i} x_{is} \le ub_i - (k - \alpha_{i-1})\ , i = 1, \dots, m\ , k \in [1, \alpha_i] \qquad (8)$$

$$0 \le x_{ik} \le d_i \quad x_{ik} \in integer\ , i = 1, \dots, m\ , k \in [1, \alpha_i] \qquad (9)$$

$$q_k \in \{0,1\}\ \ , k = 1, \dots., \bar{n} \qquad (10)$$

The objective function (4) is to maximize the sum of the cost of panel in pattern. Inequalities (5), (6), and (7) impose the cardinality constraints, the width constraints, and the height constraint, respectively. Inequalities (8) are to strengthen the bound on the x_{ik} variables (given by inequalities (9)).

2–group & 3–group cut

The second and third sub-problem are represented as follow respectively. The main concept of 2-group cutting is to divide the sheet into two parts by a vertical cut before designing pattern in the sheet. For 3-group, the sheet is divided into two parts by a vertical cut, then one of these parts is separated by a horizontal line. The mathematical model to design this pattern is proposed by Yanasse & Morabito (2008b) These models involve integer variables μ_{kh}, a_{ijkh} and L_h which denote the number of times width w_k is cut along W in sub-plate h, the number of rectangles $l_j \times w_k$ containing a piece of type i in sub-plate h, and the length of sub-plate h, respectively. The coefficient v_{ijk} represents the value of panel i (which is π_i^* when column generation is applied) containing in the rectangle $l_j \times w_k$. Both models are shown below:

2-Group Model

$$Max\ \ \sum_{h=1}^{2} \sum_{i=1}^{m} \sum_{j=1}^{J} \sum_{k=1}^{K} v_{ijk} a_{ijkh} \qquad (11)$$

Subject to:

$$\sum_{j=1}^{J} l_j \sum_{s=1}^{s_j} 2^{s-1} \beta_{jsh} \le L_h\ ,\ for\ all\ h \qquad (12)$$

$\sum_{k=1}^{K} w_k \mu_{kh} \leq W\ , \quad for\ all\ h$ (13)

$\sum_{k=1}^{m} a_{ijkh} \leq \sum_{s=1}^{s_j} 2^{s-1} f_{iksh}\ , \quad for\ all\ j,k,h$ (14)

$f_{iksh} \leq \mu_{kh},\ for\ all\ j,k,s,h$ (15)

$f_{iksh} \geq \mu_{kh} - M(1-\beta_{jsh}),\ for\ all\ j,k,s,h$ (16)

$f_{iksh} \leq \mu_{kh} - M\beta_{jsh},\ for\ all\ j,k,s,h$ (17)

$\sum_{h=1}^{2} \sum_{j=1}^{J} \sum_{k=1}^{K} a_{ijkh} \leq b_i, \quad for\ all\ i$ (18)

$0 \leq L_1 \leq \frac{L}{2}$ (19)

$\beta_{jsh} \in \{0,1\}, \mu_{kh}, a_{ijkh} \geq 0, integer, f_{iksh} \geq 0,$ (20)
$i = 1, \dots, m\ , j = 1, \dots, J, k = 1, \dots, K,$
$s = 1, \dots, s_j, h = 1,2.$

The objective (11) is to maximize the total value of the panels cut in the pattern. Constraints (12) and (13) guarantee that the panel lengths and widths do not exceed the lengths and widths of sheet respectively. Constraint (14) to (17) limit the variables a_{ijkh} to $\mu_{kh}\beta_{jsh}$. These constraints originate from linearization propose by Skogestad et al. (1997) where $\beta_{jsh} \in \{0,1\}$, M is a large number and s_j is the maximum number of bits for a binary representation of μ_{kh}. Constraints (18) refer to the availability of the panel. Constraint (19) imposes that L_1 is less than or equal to $\frac{L}{2}$ to avoid the symmetry of patterns and constraints (20) refer to the non-negativity and integrality of the variables.

3-Group Model

$Max \quad \sum_{h=1}^{3} \sum_{i=1}^{m} \sum_{j=1}^{J} \sum_{k=1}^{K} v_{ijk} a_{ijkh}$ (21)

Subject to:

$\sum_{j=1}^{J} l_j \sum_{s=1}^{s_j} 2^{s-1} \beta_{jsh} \leq L_h\ , \ for\ all\ h$ (22)

$\sum_{k=1}^{K} w_k \mu_{kh} \leq W_h\ , \quad for\ all\ h$ (23)

$\sum_{k=1}^{m} a_{ijkh} \leq \sum_{s=1}^{s_j} 2^{s-1} f_{iksh}\ , \quad for\ all\ j,k,h$ (24)

$f_{iksh} \leq \mu_{kh},\ for\ all\ j,k,s,h$ (25)

$f_{iksh} \geq \mu_{kh} - M(1-\beta_{jsh}),\ for\ all\ j,k,s,h$ (26)

$f_{iksh} \leq M\beta_{jsh},\ for\ all\ j,k,s,h$ (27)

$\sum_{h=1}^{3} \sum_{j=1}^{J} \sum_{k=1}^{K} a_{ijkh} \leq b_i, \quad for\ all\ i$ (28)

$0 \leq W_2 \leq \frac{W}{2}$ (29)

$\beta_{j\,sh} \in \{0,1\}, \mu_{kh}, a_{ijkh} \geq 0, integer, f_{iksh} \geq 0,$ (30)
$i = 1, \dots, m\ , j = 1, \dots, J\ , k = 1, \dots, K,$
$s = 1, \dots, s_j, h = 1,2,3.$

The objective (21) is to maximize the total value of the panels cut in the pattern. Constraints (22) to (28) correspond to constraints (12) to (18) for each sub-plate h. Constraint (29) imposes that W_2 is less than or equal to $\frac{W}{2}$, to avoid the symmetry of patterns and constraints (30) refer to the non-negativity and integrality of the variables. We refer the reader to Yanasse & Morabito (2008b) for more details.

For each step of column generation, 3 sub-problems are solved with the dual solution of the current optimal solution of RMP for each sheet type. If the feasible pattern with maximum objective value greater than zero is found, the column corresponding to this pattern is added to the current RMP. Otherwise, the current optimal solution of RMP is the optimal solution of MP.

Finding Integer Solution

In order to obtain the integer solution, the CPLEX solver is used to solve the Integer Linear

Programming (ILP). After the column generation is applied to the relaxation problem of RMP, we have the set of feasible patterns which are generated by the column generation. We considered the RMP with this set of patterns as ILP, meaning all decision variables corresponding to the number of times of using patterns are integer values. This strategy is shown below.

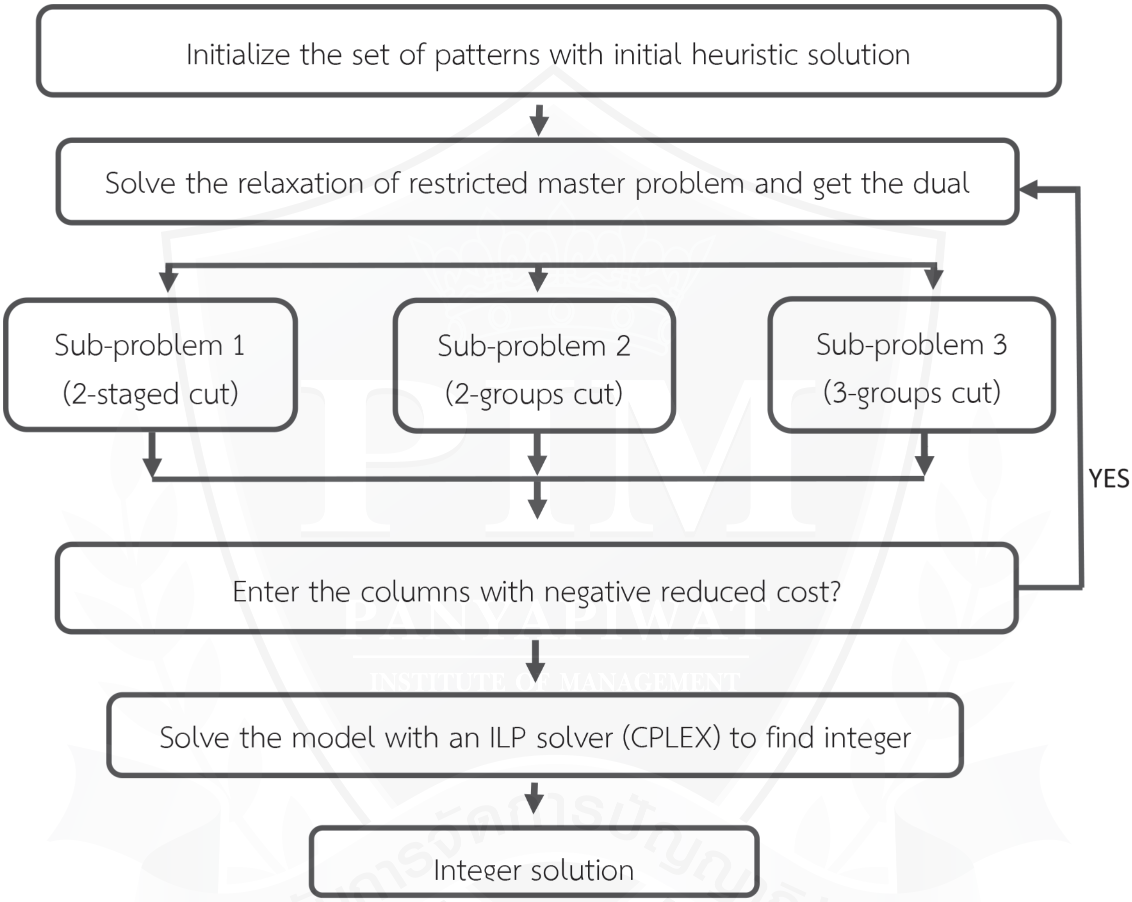


Figure 3 Algorithm for solving 2DCSP

In this section, we test the algorithm with real world data instances from electronic board industry. These instances, which are originated from monthly order of customers, are divided by grouping the sheet and the panel with the same material together. Therefore, the instances have difference numbers of sheet types, numbers of panel types, and numbers of all required panels. The algorithm described in the previous section was implemented in OPL language with CPLEX solver (version 12.6) running on Intel i7 2.50 GHz CPU, with 8 GB of memory, under windows operating system.

Table 1 Instances

Instances	Instances information										
	Sheet type (n)	W_{max}	W_{min}	L_{max}	L_{min}	Panel type (m)	w_{max}	w_{min}	l_{max}	l_{min}	All demands
1	2	51	50.5	42	42	2	25.55	25.5	20.9	20.7	219
2	2	40	36	48	48	2	20.15	20.15	24	24	301
3	2	45	40.5	43	42	2	20.5	15	21.5	21.25	575
4	3	58.5	50.5	48	42	3	26.8	19.45	24	19	537
5	3	40	24	48	48	3	22.25	19.6	22.9	22.9	552
6	4	50.5	41	48	42	2	25	25	20	20	300
7	5	51.5	37	49	42	1	14	14	25.9	25.9	209
8	5	51.5	43.5	49	48	6	25.95	16.85	24	24	1492
9	5	42	24	48	48	8	28	19.5	23	22.5	2240
10	6	51.5	41.5	49	48	2	26	20.5	24	23	1329
11	6	53.5	36	48	48	3	20.7	19	23	22	219
12	8	53	36	48	48	7	26	22.5	24	22.5	596
13	9	58	43.5	48	42	3	26.2	21.85	24	22	2192
14	9	52	24	48	48	9	22.85	16	27.45	22	3068
15	11	55.5	24	49	42	4	27	16	24	20.5	4429
16	11	57	39.5	49	42	5	26.5	20	24	22	748

Sixteen electronic board data instances are considered. The table reports the instance name in the first column. The later part of the table reports the instance information which consists of the number of sheet types, the maximum and minimum sheet widths (W_{max} and W_{min}) and lengths (L_{max} and L_{min}), the number of panel types, the maximum and minimum panel widths (w_{max} and w_{min}) and lengths (l_{max} and l_{min}), and overall demands of the panels. We compared the solution with a basic method which has been widely applied to design patterns. In this method, for each panel type, the panels are put as many as possible for all sheet types and the pattern with the least waste is chosen. The computational results are shown in the table below:

Table 2 Results

Instances	Basic method (Waste)	3 sub-model of column generation								
		With *Int*1					With *Int*2			
		Opt	col_{in}	col_s	Time (Sec)	*Gap* (%)	*Opt*	*cols*	Time (Sec)	*Gap* (%)
1	58228.9	58228.9	10	20	0.76	0	58228.9	22	1.75	0
2	115364	115364	8	14	0.62	0	115364	20	1.56	0
3	130196.7	98370.8	11	17	1.06	-24.44	98370.8	16	1.77	-24.44
4	175847.4	34377.9	28	38	2.12	-80.45	35250.9	21	2.67	-79.95
5	29108.2	29108.2	20	32	1.45	0	29761	29	3.12	2.24
6	34224	31800	20	24	1.29	-7.08	31800	16	1.86	-7.08
7	70376.6	70101.6	13	17	1.31	-0.39	70101.6	17	2.27	-0.39
8	558711.6	276556	62	82	9.41	-50.50	274640	58	10.93	-50.84
9	95692.2	94234.2	78	100	7.66	-1.52	93715.8	105	29.38	-2.06
10	351808.5	194321	36	47	8.79	-44.76	194867	32	11.89	-44.60
11	9976.9	5656.9	62	80	4.86	-43.30	7888.9	45	5.72	-20.92
12	43785	42777	164	176	5.12	-2.30	44577	65	7.55	1.80
13	508274	115340	65	93	28.07	-77.30	115340	62	33.48	-77.30
14	1270337.7	962850	198	231	30.36	-24.20	964098	161	31.26	-24.10
15	17790.5	8600.4	89	112	104.57	-51.65	8425	56	136.79	-52.64
16	51962.5	27854	130	152	10.63	-46.39	27854	88	17.25	-46.39
Average	**220105.5**	**135346.3**			**13.63**	**-28.39**	**135642.6**		**16.82**	**-26.67**

According to table 1, the first column shows the name of instances. The second column reports the waste from the basic method. Columns 3 to 7 show the results from column generation with *Int*1 initial set, namely the optimal solution (*Opt*), the number of columns in the initial set (col_{in}), the number of columns to find integer solution (cols), the overall computing time (Time) and the gap percentage corresponding to a relative difference between waste from the basic method and the waste from the column generation. The gap percentage is computed by the following formula:

$$Gap\ (\%) = \frac{waste_{column} - waste_{basic}}{waste_{basic}} \times 100$$

The corresponding information for column generation with *Int*2 initial set is reported in columns 8 to 11.

The results show that the column generation with 3 sub-problems described in previous section provided the average optimal solution (waste) of 135,346.3 (using *Int*1) and 135,642.6

(using *Int*2) while the waste from the basic method is 220,105.5 on average. The gap percentages show that the column generation with *Int*1 and *Int*2 can reduce the waste from cutting around 28.39% and 26.67%, respectively. We can notice that, in some instances, column generation with *Int*2 performed worse than column generation with *Int*1 and basic method. It may cause from the lack of appropriate patterns for integer solution finding. These results also show that the efficiency of the integer optimal solution may depend on the initial pattern set.

Conclusion

In this paper, we consider the 2DCSP with multiple sheet types when a sheet is cut through the guillotine cut. We proposed the new method based on column generation technique to solve this problem and to investigate the method when it have multiple sub-problems. For these sub-problems, patterns are designed in 2-staged cutting pattern, 2-group cutting pattern and 3-group cutting pattern to find attractive columns to the restricted master problem. The good set of initial patterns may reduce the number of generated sub-problems for optimality checking and may provide a better solution.

For further study, we will investigate the column generation method integrated with branch-and-bound algorithm, which is called branch-and-price, for solving 2DCSP. This procedure may provide other columns which cannot be generated at the root node and improve the integer optimal solution.

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