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Reputation in an Environment with Adaptive Customers

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Abstract

This paper studies the evolution of firms' reputation by in an infinite repeated game in which firms are rational but customers are adaptive. Customer behavior is not rational but adaptive and governed by the replicator dynamic equation. In this game, firms choose effort levels in order to produce quality goods. The customers' strategy is to choose the firm they would buy the good f rom. Dif f erent f rom existing studies, numerical results f rom numerical dynamic programming show that the equilibrium is unique and can replicate several real market ch aracteristics.

Keywords: Reputation, Repeated Games, Adaptive Agents

1. Introduction

Economists have long studied why firms do good things to maintain their reputation in the absence of perfect monitoring. Although reputation is ubiquitous and equilibria with reputation are generally efficient, it is difficult to explain this phenomenon in a standard rational environment under the presence of imperfect monitoring. With imperfect monitoring, each player can observe outcomes but cannot directly observe the actions of the other players. Although outcome and actions are correlated, the correlation is not perfect. For example, when a customer visits a restaurant, he can observe the quality of the food but cannot observe the effort the chef puts into making the food. The food quality depends on the chef's efforts and other random factors, such as the customer's mood. If the food tastes bad, the customer cannot be certain if it is the chef's fault.

In an imperfect monitoring environment, to maintain its reputation, a firm has to pay high costs to produce high-quality products. However, when customers have a strong belief that the firm always puts high effort, it is more profitable for the firm to put a lower effort. Given that the firm has a good reputation, when a customer observes a poor quality product from the firm, the customer would still trust in the firm and believe that the poor quality results from random factors rather the firm's low effort. Therefore, when the firm has good reputation, the reputation will be exploited and then collapse. Accordingly, reputation seems to be a paradox and cannot be sustained in a rational environment with imperfect monitoring.

To understand how reputation evolves or collapses in the context of imperfect monitoring, existing literature models reputation in infinite repeated games played by Bayesian rational firms and customers. Holstorm (1999) shows that, under imperfect monitoring, there exists no equilibrium in which a single firm always exerts high effort. In other words, firms do sometimes cheat in equilibrium. His argument is that if customers believe that a firm always exerts a high effort; it is more profitable for the firm to deviate because when customers believe with high probability that the firm always exert a high effort, a bad outcome only slightly affects his reputation and future profit.

To have equilibrium in which firms always exert high effort and maintain reputation in equilibrium, several papers assume the existence of few irrationally bad firms that always exert low efforts. Whether a firm is rational or irrational is its private information (type) and there is a small likelihood that a firm's type is switched between periods. Maliath and Samuelson (2001 and 2006) show that it is possible to support an equilibrium in which a normal firm puts a high effort under these assumptions. In this model, a customer's belief that a firm always provides high-quality goods cannot be very high because of the uncertainty about the firm's type. Therefore, the Holstorm (1999) reasoning cannot be applied in this case. Different from the previous literature that studies how a firm maintains his reputation to maximize future profit under imperfect competition, Horner (2002) studies how reputation arises under perfect competition where firms' profit is zero. Similar to the previous work, there are bad firms in his model. In his model, firms' prices vary according to its reputation. In beginning periods, firms have no reputation, set low prices and get negative profits. As time passes, firms have good histories on producing good products. Firms' profit increases and becomes positive when they have a good reputation. Although firms get negative profit in the first periods and positive profit in the following periods, the expected sum of discounted stream of profit is zero. In this model, the incentive to get positive profit in the future prevents firms from exerting a low effort.

In conclusion, in existing studies, in order to have equilibrium with reputation, a small fraction of irrational firms are necessary. However, no good reason is provided on how irrational firms exist and survive market competition. In such a setting, as a result of folk theorem (Horner and Olszewski (2008)), reputation exists only in some equilibria among infinitely many equilibria. Therefore, reputation is just one possible outcome in such games. Moreover, all of these studies consider the simplest static equilibrium with reputation in which firms always exert high fixed effort. Therefore, the equilibria studied cannot explain the dynamics of reputation observed in reality.

Departing from the existing literature, we study repeated games between firms and customers in which firms are perfectly rational but customers are boundedly rational and adaptive. Customers are not doing complicate optimization in our models; they are evolutionarily learning. It is reasonable to assume that consumers are adaptive. Existing studies (for example see Lux, 1995) show that many times people are not rational but follow some herd behavior. On the other hand, firms are likely to be rational, calculating, and use more information to maximize profit. Our numerical results suggest that the equilibrium in our model is unique. In such equilibrium, reputation arises naturally. Moreover, different from existing rational models whose equilibriums are pretty static, our models can replicate several market rich dynamic characteristics. For example, firms initially put very high effort to attract customers in the beginning and then decrease their effort to a steady state level.

The organization of the paper is as follows. Section 2 introduces our basic two period models with adaptive customers and compares our model with some standard models in the literature. In section 3 and 4, we study the model with monopoly and duopoly, respectively. Section 5 concludes.

2. The Two-period Models

In this section, to get basic understanding of how our adaptive models are different from those standard models in the literature, we explore and compare three models in a simple dynamic environment: the standard model, the standard model with some irrational types and our model with adaptive customers. In the first two models, the equilibrium in the presence of perfect monitoring and imperfect monitoring is qualitatively the same. Therefore, for the sake of simplicity, we assume perfect monitoring in the first two models. Imperfect monitoring is simply introduced in section 2.3.

2.1 The Standard Model

In this section, we consider a standard two-period game between a monopoly and a continuum of customers with size 1. The price is assumed to be fixed and equal to 1 in the two periods. In each period, the monopoly chooses an effort level $e \in [0,10]$. An e-level effort produces a unit of goods with quality e. The cost of effort e is ce, $c \in (0,1)$. The cost of effort only incurs when goods are sold.

Each consumer has two options in each period: to buy one unit of goods from the monopoly or don't buy and spend his money on the other goods. We assume that customers cannot observe product quality before buying it. As a normalization, the consumer's net utility after buying the monopoly's good with effort e is e. Spending money on the other goods yields 1 unit of utility. Spending on the monopoly's goods gives the customer e unit of utility.

The expect payoff of each stage game can be summarized as follows:

$$u(\text{buy}) = e, \quad u(\text{don't buy}) = 1.$$

where u is the customer's utility and {buy, don't buy} is the customer's choice. For the monopoly, his profit for each unit of good sold is $1 - ce$.

Obviously in the stage game, the dominant strategy for the monopoly is $e = 0$. Therefore, the consumer's best response is don't buy. The Nash equilibrium is {$e = 0$, don't buy}. In this equilibrium, the monopoly excerts 0 effort. It is easy to see that in the two period game, the subgame perfect equilibrium outcome is similar to that in the stage game. In this equilibrium, the firm puts the lowest effort in both periods and customers do not buy from the firm in both periods. Therefore, there is no reputation arise as an outcome of this game.

2.2 The Standard Model with a Commitment Type

In this section, as widely used in the literature, we add uncertainty on the type of the monopoly to the model in order to show that reputation is a possible outcome in such model. With probability p, the monopoly is *irrationally* benevolent and always exerts $e = 10$ in both periods. With probability 1-p, the monopoly is rational and maximizing profit. The probability p can be arbitrarily small. In this model, with $c = 0.01$, we claim that a perfect Bayesian equilibrium is following. The maximizing-profit monopoly employs $e = 10$ in the first period and $e = 0$ in the second period. The consumer's strategy is: buy in the first period; and buy in the second period if in the first period he observe e = 10 in the first period. To verify the equilibrium claim, we have to show that the customer's and the monopoly's strategies are optimal.

We first verify that the strategy of the customer is optimal. Fixing the monopoly's strategy, the total expected utility from the equilibrium strategy from the whole game is $p(10+10) + (1-p)(10+0) = 10p + 10$. If the customer deviates to *don't buy* and spends money on the other good in the first period, his expected utility is (1+1). Clearly, the customer is employing the best strategy. We now verify that the monopoly's strategy is optimal. If the monopoly would like to deviate from the current strategy, his best deviation is to put $e = 0$. By putting zero effort, the monopoly would get 1 unit of profit in the first period and no profit in the last period. His total profit is then 1. Under the equilibrium strategy, the monopoly gets 0.9 unit of profit in the first period and 1 unit of profit in the second period. Clearly, the monopoly's strategy in the equilibrium is optimal.

In the perfect Bayesian equilibrium proposed above, we observe that the monopoly put high effort in the first period in order to build his reputation. However, in the last period, the reputation collapsed. However, in an infinitely repeated version of this game, there is no last period and there exists an equilibrium in which reputation can be sustained for good.

2.3 The Model with Adaptive Consumers

In this section, we introduce our main basic model: the two-period model with adaptive customers. This model will be extended in an infinite period setting in the subsequent section. The monopoly in this model is similar to that in section 2.1. The monopoly is always rational and chooses his effort levels in order to maximize profit. However, in this model, the effort and the quality of the good are not perfectly correlated. Because consumers only observe quality and not the effort, imperfect monitoring is present in this model. The monopoly with an e-level effort produces a unit of goods with quality $e + \xi$. Where ξ is some random variable with zero mean with full support. Therefore, on average the monopoly with effort e produces goods with quality e.

Customers are adaptive; they employ an adaptive learning process based on the replicator dynamics equation. Replicator dynamics are based on the premise that the fraction of a strategy used by the population in the next period depends on the total payoff of the population who employs that strategy relative to the total payoff of the population in the current period. More details on replicator dynamics and adaptive learning can be found in Samuelson (1997) and Levine and Fudenberg (1998). Customer behavior in our model is governed by the following replicator dynamics equation:

$$n_2 = \frac{n_1 e_1}{n_1 e_1 + (1 - n_1)}$$

where n_t and e_t are respectively the number of the monopoly's customer and the monopoly's effort level in period t.[2] $n_1 e_1$ is the aggregate payoff of the customer buying from the monopoly. $(1–n_1)$ is the aggregate payoff of the consumers who buy the other goods. Consequently, $n_1 e_1 + (1–n_1)$ is the total payoff of all customers in period 1. In this model, we assume that n_1 is assumed to be an exogenous.

Knowing that customers are adaptive, the monopoly's problem is as follow:

$$\max_{e_1, e_2} (1 - ce_1)n_1 + (1 - ce_2)n_2 \text{ s.t. } \quad n_2 = \frac{n_1 e_1}{n_1 e_1 + (1 - n_1).1}$$

Assuming interior solution exists, the optimal effort for the monopoly in period 1 is

$$e_1 = \frac{\sqrt{c(1 - n_1)} - (1 - n_1)}{cn_1} \text{ and } e_2 = 0.$$

In this model, the monopoly exploits customers' adaptive behavior by building his reputation in the first period and abusing it in the second period. Although our model produces similar firm behavior as the rational model with irrationally benevolent firms in section 2.2 does, consumer behavior is different. In our model, the monopoly has more customers in the second period but in the rational model, the number of customers the firm has in each period 2 (n_2) can be either more or less than n_1.

In this section, we have shown that our model with adaptive consumers can generate equilibrium outcome with some reputation as in the model with commitment types. The next section will extend this basic model in a more general setting.

3. Infinite Repeated Games with Adaptive Consumers

In this section, we extend the model in section 2.3 in an infinitely repeated setting. We focus on Markov's equilibrium in which the firm's strategy in each period depends only on the number of customers in that period. The firm's optimization problem can be defined as the following dynamic programming problem:

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[2] From the replicator dynamics equation, it seems that perfect monitoring on the average quality is required for the total payoff of all consumers. However, without perfect monitoring, the aggregate behavior can be generated by an individual behavior similar to the following individual behavior: each consumer compares his utility with another random consumer utility in each period. If the utility is lower, he imitates the strategy of the other customer in the next period.

$$V(n_t) = \max_{e_t} (1 - ce_t)n_t + \beta V(n_{t+1})$$

$$n_{t+1} \qquad \frac{n_t e_t}{n_t e_t + (1 - n_t)} \tag{3.2}$$

where V is the value function of the firm, $(1- ce_t)n_t$ is the firm's payoff in period t. As a common practice, to make the total payoff of the game finite, we introduce the discount factor $\beta \in (0,1)$. It is easy to show that the value function that satisfies (3.1) is unique by applying Blackwell's sufficient conditions followed by the contraction-mapping theorem as shown in Stokey and Lucas (1989).

Although the model is simple, its closed form solution is not available. Therefore, numerical dynamic programming methods are employed. Equation (3.1) was numerically solved by the discretized state space method with 1000 grids as suggested by Judd (1997, chapter 12). The algorithm can be summarized as follow

Step 1: Set $n = 0$. Make initial guess V_0

Step 2: Find $V_{n+1} = \max_{e_t} (1\text{-}ce_t)n_t + \beta V_n$

Step 3: Check whether $\| V_{n+1} - V_n \|$ is sufficiently small and V converges or not

Step 3.1: if yes, V_{n+1} is the solution to the Bellman equation (3.1)

Step 3.2: if no, increase n by 1 and go to Step 2.

The model is numerically solved under the following parameter set: $\beta = 0.9$, $c = 0.5$.

Figure 3.1 shows the value function solved by the numerical method above implemented by the *C* programming language. As expected, the value function is increasing in n. Because the number of customers represents the firm's reputation, this can be interpreted as the value of the firm increasing in its reputation. After obtaining the value function V, we can numerically solve the policy function $e(.)$ from (3.1). Figure 3.2 shows the policy function e as a function of n. We observe that the effort level is decreasing in the number of customers.

The number of the next period customers, n_{t+1}, is plotted with the number of current period customers, n_t, in Figure 3.3. The plot shows that the number of customers in the next period is increasing in the number of customer in this period. The straight line in this figure is the 45-degree line. This graph suggests that the stable steady state value of n is unique.

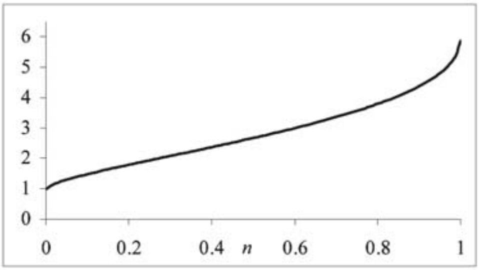

Figure 3.1: Value Function

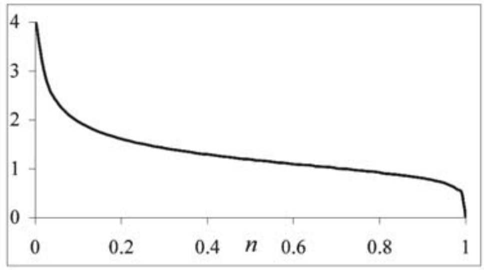

Figure 3.2: Effort Levels

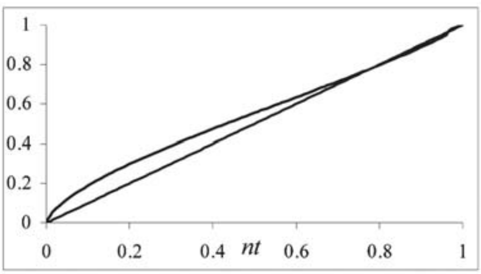

Figure 3.3: n_{t+1}

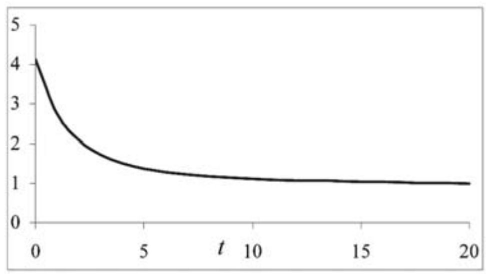

Figure 3.4: Effort Levels

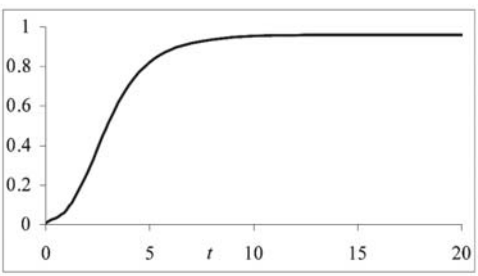

Figure 3.5: Number of Customers

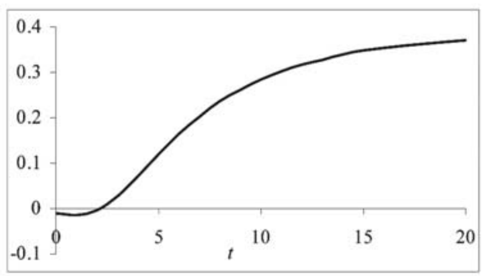

Figure 3.6: Profit

Figures 3.4-3.6 show the simulated time paths of the firm's effort levels, customers and profit. The simulation assumes that initially the number of customers the firm has is 0.01 ($n_1 = 0.01$). Figure 3.4 shows that the firm exerts very high efforts during the first periods. Figure 3.5 shows that the number of customers increase and become a constant. Figure 5.6 shows that the firm's profit is *negative* in the first periods and become a positive constant. This is consistent with real world observation that firms invest in advertising and promotion in order to introduce new products to markets. Various sets of parameter values are used for sensitivity analysis. Similar results are obtained when c is low and β is high. If c, the marginal cost of efforts, is too high or β, the discounting factor, is too low, the firm's optimal strategy is to put the lowest effort forever and reputation does not arise.

Now, we turn to analytical analysis of the steady state values n and e. Obviously in the steady state, to have $n_{t+1} = n_t$ we need $e_t = 1$. We are to solve the steady state value of n in this game. Rewriting (3.2), we have

$$e_t = \frac{n_{t+1}(1-n_t)}{n_t(1-n_{t+1})} \tag{3.3}$$

Substituting this equation into (3.1) and differentiating with respect to n_t using the envelop theorem, we obtain

$$V'(n_t) = 1 + \frac{cn_t}{1-n_{t+1}} \tag{3.4}$$

Substituting (3.3) in the left hand side of (3.1) and differentiating with respect to n^1_{t+1}, we obtain the following first order condition for problem (3.1):

$$-\frac{c(1-n_t)}{(1-n_{t+1})^2} + \beta V'(n_{t+1}) = 0 \tag{3.5}$$

Using the fact that at the state, $n_t = n_{t+1} = n^*$, substituting (3.4) into (3.5) and solving for n^*, we have $n^* = \frac{\beta - c}{\beta(1-c)}$. It is straight forward to show that $\frac{dn^*}{d\beta} > 0$ and . $\frac{dn^*}{dc} < 0$. The number of monopoly customers in the steady state is increasing in the discount factor and decreasing in its unit cost. The higher the discount factor, the more patient the monopoly is. The monopoly can use more aggressive policy and bear negative profit in the beginning to attract customers in the steady state.

4. Dynamic Duopoly Models

We now extend the previous model by adding another firm to the game. In this extended model, customers have two strategies: buying from firm 1 or buying from firm 2. Customers no longer have the option to spend their money on other goods. Firms 1 and 2 are identical to the monopoly in the previous section and competing with each other. The strategy of each firm is his effort level. Aggregate customers' behavior is governed by the following replicator dynamic equation:

$$n_{t+1}^{i} = \frac{n_t^i e_t^i}{n_t^i e_t^i + (1-n_t^i)e_t^{-i}}$$

where superscripts represent firms and subscripts represent periods. n_t^i is the number of customers of firm *i* in period *t* and e_t^i is the effort level of firm *i* in period *t*.

We consider only the symmetric Markov equilibria, where a firm's strategy depends only on its number of customers and the strategy of each firm is identical. Assuming a Markov equilibrium, firm *i*'s problems can be summarized by the following dynamic program:

$$V^i(n_t^i) = \max_{e^1 \in [0,\bar{e}]} (1 - ce_t^i)n_t^i + \beta V^{-i}(n_{t+1}^i) \quad \text{for } i = 1,2 \tag{4.1}$$

$$\text{s.t } n_{t+1}^{i} = \frac{n_t^i e_t^i}{n_t^i e_t^i + (1-n_t^i)e^{-i^*}(n_t^{-i})} \tag{4.2}$$

Because each consumer must buy from either firm 1 or firm 2, $n_t^1 + n_t^2 = 1$. The term $e^{-i^*}(n_t^{-i})$ is the strategy (effort level) of firm $-i$ as a function of n_t^{-i}. A symmetric Nash equilibrium requires that $V^1(.) = V^2(.)$ and $e^{1*}(.) = e^{2*}(.)$.

To solve for a symmetric Markov Nash equilibrium, the numerical method for solving fixed points suggested by Miranda and Fackler (2002) is applied. The method can be summarized as followed.

Step 1: Set $n = 0$. Make initial guess for the function e^{2*}.

Step 2: Fixing e^{2*}, find policy function e^{1*} that solve equation (4.1).

Step 3: Check whether $\| e^{1*} - e^{2*} \|$ is sufficiently small or not

Step 3.1: if yes, e^{1*} is the symmetric Nash equilibrium strategy

Step 3.1: if no, replace e^{1*} with e^{2*} and go to Step 2.

Note that this algorithm requires that the Nash equilibrium is locally stable around the initial guess. The following parameters, $c = 1$, $\beta = 0.9$ are used. Because the equilibria found by this algorithm depends on initial guess, 1000 different random initial guesses were used. These 1000 random guesses converge to the same equilibrium. This result suggests that the symmetric Markov equilibrium is unique.

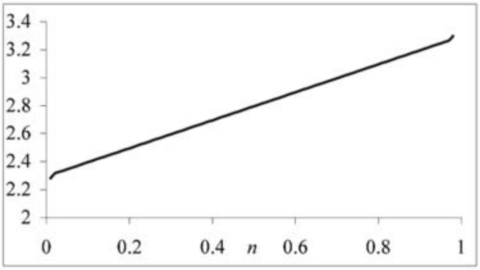

Figure 4.1: The Value Function

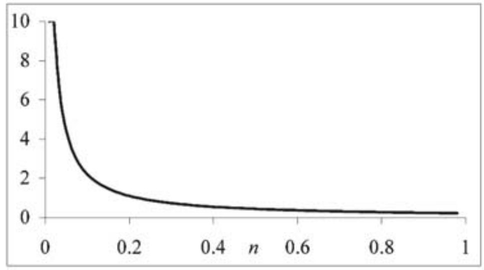

Figure 4.2: Effort Levels

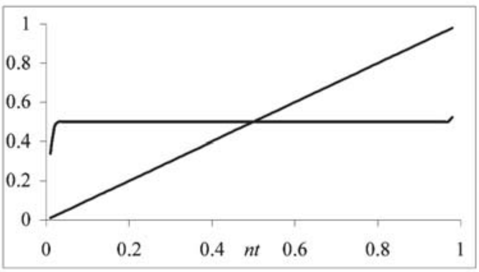

Figure 4.3: n_{t+1}

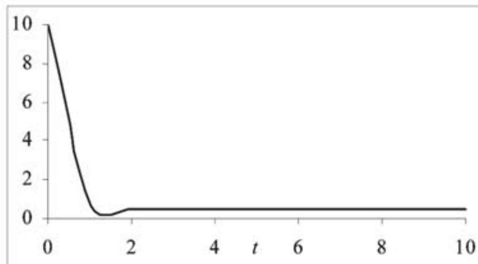

Figure 4.4: Firm 1's Effort Levels

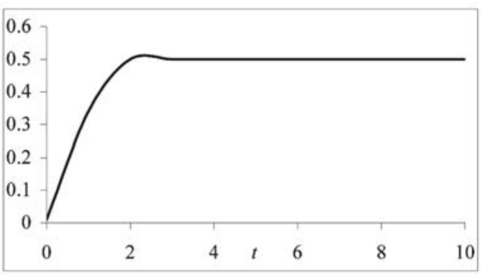

Figure 4.5: Firm 1's Customers

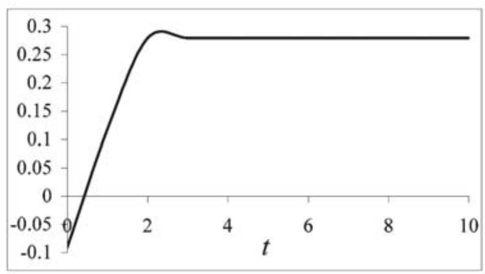

Figure 4.5: Firm 1's Profits

Figures 4.1-4.3 show the value function, the policy function and n_{t+1}^{i} respectively. Similar to the monopoly case, the value function is increasing in n and the policy function is decreasing in *n*.

Figures 4.4-4.6 show the simulated time path of firm 1's effort level, number of customer and profit under the initial environment with n_1^1 is 0.01 and n_1^2 is 0.99. In this simulation, firm 1 is the new entrant and firm 2 is the incumbent in the market. From these figures, we see that firm 1 puts very high effort and has a negative profit to attract consumers in the first periods. Then the effort level and the profit, respectively, decrease and increase over time. In the steady state we find the two firms put the same effort level and have equal market shares.

Similar to what we did in the previous section, we now analytically solve the steady state value of *e* and *n*. Because the two firms, equally share the customers, the value of *n* in the symmetric steady state is 0.5. To solve for *e*, we first rearrange equation (4.2) and have

$$e_t^i = \frac{n_{t+1}^i e_t^{-i}(1-n_t^i)}{n_t^i(1-n_{t+1}^i)} \tag{4.3}$$

Plugging this equation into (4.1), from the envelop theorem, we have

$$V^{/}(n_t^i) = 1 + \frac{cn_t^i e_t^{-i}}{1-n_{t+1}^i} \tag{4.4}$$

Substituting (4.3) in (4.1) and differentiating with respect to n_{t+1}^i , we obtain the following first order condition for the optimization problem (4.1):

$$-\frac{ce_t^{-i}(1-n_t^i)}{(1-n_{t+1}^i)^2} + \beta V^{/}(n_{t+1}^i) = 0 \tag{4.5}$$

Using the fact that at the state $n_i^{-i} = n_{t+1}^{-i} = n^* = 0.5$, $e_t^1 = e_t^2 = e^*$, substituting (4.4) into (4.5) and solving for e^* we have $e^* = \frac{\beta}{c(2-\beta)}$ and $\frac{de^*}{d\beta} > 0$ and $\frac{de^*}{dc} < 0$. The steady state effort level is increasing in the discount factor and decreasing in the unit cost.

5. Conclusion

In this paper, we have developed a model to study the dynamic of reputation in the presence of adaptive consumers. We have shown numerically how reputation naturally emerges in an infinitely repeated game where customers are adaptive but firms are rational. The models with a monopoly and duopoly are studied. Results from both models are consistent. Different from existing models, in which there are infinitely many equilibria, our models generate unique equilibria. Moreover, our models are successful in replicating dynamic stylized facts that could not be generated by previous standard models. Interesting stylized facts generated by our models are as follows. New entrants work hard to produce high quality goods and build their reputation to attract customers. When firms have more customers, their effort level is reduced to some positive value and their reputation is maintained in the steady state. We also find that firms' efforts in the steady state increases in the discount factor.

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