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Bubbles in a World Asset: The Case of Cryptocurrency

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Abstract

This paper explores the rational bubbles in cryptocurrency, an international asset with fixed supply and negligible transaction cost, using a simple macroeconomic model. We show that cryptocurrency exhibits oscillatory and volatile dynamics under certain conditions. We also analyze the welfare and spillover effects of cryptocurrency for different parameter values. Cryptocurrencies may increase welfare for agents in economies with certain parameters, such as those with low relative risk aversion or high output elasticity of capital. Cryptocurrency can easily transfer shocks from one country to another through pricing channels or through propagated risk perception. Our paper contributes to the scarce literature on cryptocurrency in macroeconomic contexts.

Keywords: cryptocurrency, bitcoin, bubble, welfare, contagion.

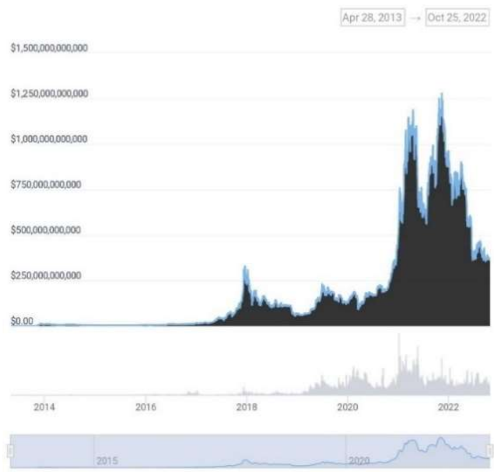
1. Introduction

Although cryptocurrencies are no longer new to the world, literature on cryptocurrencies, especially in the context of macroeconomic models, is very limited. This paper aims to contribute to the existing literature on cryptocurrencies by modeling them in an overlapping generations (OLG) model, studying their impact on welfare (consumption) and their role as a contagion of economic shocks.

Cryptocurrency bubbles have nearly identical prices across markets around the world, which is different from traditional bubbly assets such as real estate or fiat money, which have high transaction costs and vary in value across borders. A cryptocurrency is a digital asset or currency secured by cryptography, making it very difficult for anyone to create counterfeit versions or double-spend them. A key feature of cryptocurrencies is that they are not issued by a government or central authority such as a national bank. Governments, therefore, find it difficult to manipulate or intervene in using and transferring cryptocurrencies. The word crypto refers to different types of encryption algorithms that protect the integrity of the asset. Due to the high volatility of the prices of these cryptocurrencies, they are not often used for retail purposes. Speculation, black market purchases, cross-border transfers, and stores of value are usually more common uses of cryptocurrencies.

The market capitalization, or the total value of all bitcoins, reached a high of more than 1,250 billion USD in late 2021, which is comparable to the market capitalization of smaller stock exchanges and more than double the market capitalization of the Stock Exchange of Thailand at about 540 billion USD (at 37 THB/USD) in March 2022 (www.set.or.th). Figure 1 shows the market capitalization of bitcoin from 2014 to late 2022.

Figure ***Error! No text of specified style in document.***. BTC market capitalization



Notes: Bitcoin price in USD, BTC Live Price Chart & News (CoinGecko)

It is evident that the market capitalization growth of bitcoin is driven by its price, which surged beginning in the middle of 2017 and peaked at the end of the same year. This surprising price appreciation is due to the firm belief that cryptocurrency, with its transparency in terms of amount in circulation, low transaction fees, and security, will be able to replace fiat currencies. However, as cryptocurrency gained momentum, several governments and official financial regulators around the world expressed concern and even banned the use or trading of cryptocurrency. People lost that belief, and the price dropped significantly in the following year. Bitcoin price surged again in 2021 behind several events, including a $69 million sale of a non-fungible token (NFT), a type of cryptocurrency, and Elon Musk contributing to the record high price of dogecoin (DOGE).

2. Related Literature

To make the claim that cryptocurrency is a bubble, Cheah and Fry (2015) developed an asset pricing model to try to derive the intrinsic value of cryptocurrency. They found that the intrinsic value of cryptocurrency is zero, so to have a positive price at all signals, a bubble forms on this asset. Empirical results show that the size of cryptocurrency bubbles fluctuated heavily over the course of three years, mostly from price speculation and overall investment sentiment. Su et al. (2018) tested for multiple bubbles in cryptocurrency markets and associated surges in cryptocurrency prices worldwide with economic crises in any given country. As people lose confidence in their quickly depreciating national currencies, they try to find a haven that is not linked to the performance of their economies. Cryptocurrency was the answer, and through the convenient acquisition of cryptocurrency worldwide, the sharp increase in demand for cryptocurrency in distressed countries drove cryptocurrency prices up everywhere else through arbitrage activities. This evidence shows that cryptocurrency by itself does not have a fundamental value, but it is through price speculation and coordination belief that drive up the price and make it a bubble.

Regarding the impact of cryptocurrency bubbles, Böhme et al. (2015) contrasted cryptocurrency's nature of limited supply to that of excessive growth of money supply. As opposed to the inflationary pressure imposed on prices of goods when the money supply grows faster than demand, cryptocurrency's restricted supply can cause a deflationary economic crisis should it be used widely. At present, there is limited study into the effect of cryptocurrency on the economy using macroeconomic indicators such as GDP.

Although many papers have been published in recent years on cryptocurrencies and bitcoin since they gained popularity, most papers try to fit bitcoin into the definition of economic bubbles and identify where cryptocurrency bubbles have occurred in the world. Few papers actually study cryptocurrencies in the eye of classic economic growth models or examine how bitcoin or cryptocurrencies affect agents' welfare and the contagions of economic shocks between countries. We present a review of some related literature here. Haykir and Yagli (2022) investigated speculative bubbles in cryptocurrencies and factors impacting the bubbles during the COVID-19 pandemic. They concluded that all the cryptocurrencies covered exhibited bubble traits and that explosive behavior in one cryptocurrency led to another during the pandemic but not during the bubble stage. This implies that a cryptocurrency bubble is not explained by herd behavior. Market-specific factors such as Google trends and volume are positively associated with speculative bubble prediction. Gorse (2017) used epidemic detection techniques to apply the same technique to social media data to predict cryptocurrency price bubbles and found evidence that bubbles exhibited a social epidemic-like spread of investment ideas. Association between trading volumes relating to studied cryptocurrencies and community-based social media usage was found using a hidden Markov methodology (HMM). Chen et al. (2019) examined the impact of investor sentiment on aggregate return prediction for cryptocurrencies. With a local momentum autoregression model, they found that the sentiment effect is prominent throughout the bubble period, and a reversal effect is shown once the bubble collapses. They emphasized that measuring the impact of investor sentiment for a novel asset such as cryptocurrencies is conditional on bubble regimes. Dong et al. (2022) is one of the few papers to study bitcoin with a macroeconomic model that considers bitcoin to be a risky and costly bubble. They characterize the bubbles as investments that increase liquidity for financially constrained firms while competing for real investments to fund mining capabilities, two opposing channels dominating one another under different economic parameters. In Dong et al.'s (2022) paper, bitcoin exists in a one-country model, and the emphasis is on the investment allocation decisions of households on either the capital or bitcoin market contingent on the bitcoin sentiment and idiosyncratic investment distortion shocks on the capital market. They found that depending on the initial sentiments on bitcoin, a further deterioration of sentiment may stimulate or depress the real economy depending on which channel dominates. The deterioration of bitcoin sentiment stimulates a relatively optimistic economy, while a relatively pessimistic economy is depressed.

The contribution of this paper to the related literature is to model cryptocurrencies, or more specifically, the characteristics of bitcoin, into an economic growth model.[1] We not only define the world parameters in which bubbly equilibriums may exist but also how bubbles lead to increased or decreased welfare and how cryptocurrency bubbles play a role in transmitting economic shocks across borders. We extend a two-country model because cryptocurrency is a borderless asset with negligible transaction costs across borders. Various economic factors distinguish each country's decision-making. Due to information and financial globalization, agents in each country respond to both domestic and foreign events that affect them. We employ the OLG model to capture the intertemporal and intergenerational dynamics of behavior and policies. It also incorporates the population's features and their implications for the economy and policy effectiveness. We simplify the model by assuming identical populations in both countries, but we suggest the OLG model as a promising framework for future

[1] For other related studies on this topic, the reader is directed to Kantaphayao and Sukcharoesin (2021) and Dumrongwong (2021).

research since infinite lifetime horizon models do not reflect the intergenerational dynamics in reality.

3. World Economy With Both Cryptocurrency Bubbles and Capital

We assume a two-country endowment economy with identical preference: $\beta u(c_{2t+1,i})$, where $i = 1, 2$ refers to Country 1 and Country 2, and the instantaneous utility function is strictly concave.[2] The population size of country i is denoted as N_i with no growth. We follow the Overlapping Generations Model (OLG) in which, in each period, there is an old generation and a young generation. The young in country i receive an endowment of e_i. There is an asset called a cryptocurrency bubble, which is traded internationally with no transaction cost, has a fixed positive and infinitely divisible global supply M, and pays no dividend. In period t, the young choose between consumption and buying a cryptocurrency bubble $m_{t,i}$ at price p_t. In the next period, the old sells the bubble at price p_{t+1} and consumes all of his resources. The price of a bubble is identical in both countries as the agents in each country have equal accessibility to the same cryptocurrency market. Due to the characteristics of cryptocurrency trading, cross-border frictions can be safely assumed as zero, and any difference in price occurring between exchanges in different countries can be quickly corrected through arbitrage activities. As such, we shall not include the country subscript on the price variable. Denote $m_{t,i}$ as bubble demand at time *t* of the agent in country i. For simplicity, we will assume that agents only consume during old age. Our maximization problem becomes

[2] Identical preference is not a necessary assumption. Using CRRA utility with different relative risk aversion coefficients across countries does not affect our results.

$$\max \beta E u(c_{2t+1,i}) \tag{1}$$

such that

$$e_i = P_t m_{t,i} + k_{t+1,i} \tag{2}$$

$$c_{2t+1,i} = P_{t+1} m_{t,i} + f(k_{t+1,i}). \tag{3}$$

The perceived probability of a bubble bursting is governed by a Markov process.

Table 1. Markov process governing the price of bubbles

	P_t>0	**P_t=0**
P_{t+1}>0	1-π_i	0
P_{t+1}=0	π_i	1

$k_{t+1,i}$ is the capital bought and invested when agents are young, which yields a return of $f(k_{t+1,i})$ when agents are old. Therefore, in this case, expected utility maximization becomes

$$\max \beta[(1-\pi_i)u(c_{2t+1,i,b}) + \pi_i u(c_{2t+1,i,nb})] \tag{4}$$

where $c_{2t+1,i,b}$ refers to the consumption of agents in country *i* when there is a bubble, and $c_{2t+1,i,nb}$ refers to the consumption of agents in country *i* when the bubble bursts. As such:

$$c_{2t+1,i,b} = \frac{P_{t+1}}{P_t}(e_i - k_{t+1,i}) + f(k_{t+1,i}) \tag{5}$$

$$c_{2t+1,i,nb} = f(k_{t+1,i}). \tag{6}$$

The first order condition is

$$(1-\pi_i)u'(c_{2t+1,i,b})\left(\frac{P_{t+1}}{P_t} - f'(k_{t+1,i})\right) = \pi_i u'(c_{2t+1,i,nb})\left(f'(k_{t+1,i})\right). \tag{7}$$

The market clearing condition is

$$N_1 m_{t,1} + N_2 m_{t,2} = M. \tag{8}$$

Equation (7) for the two countries and (8) form the equilibrium system of equations.

3.1 Assumption 1

We assume that for both countries, a) the utility function has a continuous first and second derivations and is strictly concave, b) Inada conditions hold: $\lim_{c\to\infty} u'(c) = 0 \; and \; \lim_{c\to 0} u'(c) = \infty$, and c) the production function satisfies $f(0) = 0, f'(k) > 0 \; and \; f''(k) < 0 \; for \; all \; k > 0 \; and \; that \; \lim_{k\to\infty} f'(k) = 0 \; and \; \lim_{k\to 0} f'(k) = \infty$.

<u>Proposition 1</u>

Under Assumption 1, a unique bubbly steady state exists $\{\bar{p}, \bar{k}_i\}$, where $\bar{p} > 0$ if $f'(e_i) < 1 - \pi_i$ for $i = 1,2$.

<u>Proof</u>

The competitive equilibrium is the combination of $\{P_{t+1}, k_{t+1,1}, k_{t+1,2}\}_{t=0}^{+\infty}$

that solves $(1-\pi_i)u^{\prime\left(\frac{P_{t+1}}{P_t}(e_i - k_{t+1,i}) + f(k_{t+1,i})\right)\left(\frac{P_{t+1}}{P_t} - f'(k_{t+1,i})\right)} = \pi_i u'\left(f(k_{t+1,i})\right)\left(f'(k_{t+1,i})\right)$

and $\sum_{i=1}^{2} N_i \frac{(e_i - k_{t+1,i})}{\bar{P}} = M$ simultaneously, provided that the endowments $\{e_{t,1}, e_{t,2}\}_{t=0}^{\infty}$, initial bubble price P_0, and the constants N_1, N_2, and M are given.

At a steady state, $P_t = P_{t+1} = \bar{P}$ and $k_{t,i} = k_{t+1,i} = \bar{k}_i$ for i ∈{1,2},

such that the combination of $\bar{P}, \bar{k}_1,$ and $\bar{k}_2$ makes the first-order conditions and market clearing condition hold $(1-\pi_i)u'\left((e_i-\bar{k}_i)+f(\bar{k}_i)\right)\left(1-f'(\bar{k}_i)\right) = \pi_i u'\left(f(\bar{k}_i)\right)\left(f'(\bar{k}_i)\right)$, ∀i $\sum_{i=1}^{2} N_i \frac{(e_i-\bar{k}_i)}{\bar{P}} = M$. The steady-state FOC can be rewritten as $\frac{u'(f(\bar{k}_i))}{u'((e_i-\bar{k}_i)+f(\bar{k}_i))} = \frac{(1-\pi_i)}{\pi_i}\left(\frac{1}{f'(\bar{k}_i)}-1\right)$, $k_i \in (0, e_i)$.

According to Assumption 1, if $\frac{u'(f(\bar{k}_i))}{u'((e_i-\bar{k}_i)+f(\bar{k}_i))}$ is a continuously strictly decreasing function ranging from ∞ to 1 and $\frac{(1-\pi_i)}{\pi_i}\left(\frac{1}{f'(\bar{k}_i)}-1\right)$ is a continuously strictly increasing function ranging from $-\frac{(1-\pi_i)}{\pi_i}$ to $\frac{(1-\pi_i)}{\pi_i}\left(\frac{1}{f'(e_i)}-1\right)$, then the unique steady states $\bar{k}_i$ exists if and only if $\frac{(1-\pi_i)}{\pi_i}\left(\frac{1}{f'(e_i)}-1\right) > 1$ or $f'(e_i) < 1-\pi_i$. QED.

Proposition 1 suggests that when the return on investment in the bubbleless world (marginal product of capital when we invest all the endowment) is too low, the bubbly cryptocurrency can exist. Alternatively, we can interpret that when all people think that the probability of the bubble bursting is not too high (less than $1-f'(e_i)$), people agree on trading this common bubble worldwide.

<u>Dynamic simulation and parameter characterization</u>

Now that we have proven that with the assumptions of characteristics of utility and production functions, bubbly equilibrium can exist within a production economy when the marginal product of capital is lower than the probability that the bubble will persist. We can continue to define the characteristics of such bubbly equilibrium.

Linearizing equation (7) around the steady state yields

$$\left(k_{t+1,i}-\bar{k}_i\right) = \frac{(P_{t+1}-\bar{P})-(P_t-\bar{P})}{B_i} \quad (9)$$

where B_i is

$$\frac{\pi_i[u'(c_{2t+1,i,nb})f''(\bar{k}_i)+f'(\bar{k}_i)^2u''(c_{2t+1,i,b})]+(1-\pi_i)\left[u'(c_{2t+1,i,b})(f'\,(\bar{k}_i))+(1-f'(\bar{k}_i))^2u''(c_{2t+1,i,b})\right]}{\frac{(i-\pi_i)}{\bar{P}}[u'(c_{2t+1,i,b})+(1-f'(\bar{k}_i))u''(c_{2t+1,i,b})(e_i-\bar{k}_i)]}. \quad (10)$$

Combining equation (10) for two countries and the market clearing condition (8) yields[3]

$$(P_{t+1}-\bar{P})=\left(1-\frac{MB_1B_2}{B_1N_2+B_2N_1}\right)(P_t-\bar{P}). \quad (11)$$

Equation (11) explains the movement relationship between the price of the cryptocurrency bubble over time with the $\left(1-\frac{MB_1B_2}{B_1N_2+B_2N_1}\right)$ term determining the stability of cryptocurrency bubbles in a steady state. The steady state is considered a stable equilibrium when the term $\left(1-\frac{\mathrm{MB_1B_2}}{\mathrm{B_1N_2+B_2N_1}}\right)$ has a value between -1 and 1.

To solve this problem analytically without assuming specific utility and production functions, we need to examine the properties of the equation $\left(1-\frac{MB_1B_2}{B_1N_2+B_2N_1}\right)$. Note that B_1 and B_2 are derived from equation (11), which is quite complex and involves utility and production functions for which we do not have specific forms. Without specifics on the utility and production functions, it is difficult to determine the exact conditions under which the expression will lie between -1 and 1.

To determine the characteristics of the equilibrium numerically, we must assume an explicit utility function and specify a production function. For simplicity, we will use CRRA utility and assume a Cobb-Douglas production function (Cobb & Douglas, 1928).

[3] See Appendix I for the analytic derivation of equation (11).

3.2 Assumption 2

We assume that for both countries, the instantaneous utility and the production functions are of the forms

$$\text{a)}\quad u(c) = \beta(1-\pi)\frac{c^{1-\theta}}{1-\theta} \tag{12}$$

$$\text{b)}\quad f(k) = Ak^{\alpha} \tag{13}$$

where θ is the relative risk aversion coefficient, *A* is the total factor productivity, and α is the output elasticity of capital. Recall from Proposition 1 that in our economy with both world bubble and capital production, bubbly equilibrium exists only if the marginal product of capital is less than the probability of the bubble persisting into the next period. So, from the assumption of Cobb-Douglas production specified by equation (13), α has an impact on whether equilibrium exists under certain parameters of the countries. For this analysis, we will assume the following set of values for the parameters as the base equation. We will then vary each parameter to see the impact on the equilibrium. Note that for simplicity, we will assume that both countries are identical.[4]

Table 2. Parameter set for the base model

Parameter	α	θ	A	N	e	π	M
Value	0.4	1.5	2	2,000	400	0.2	20,000

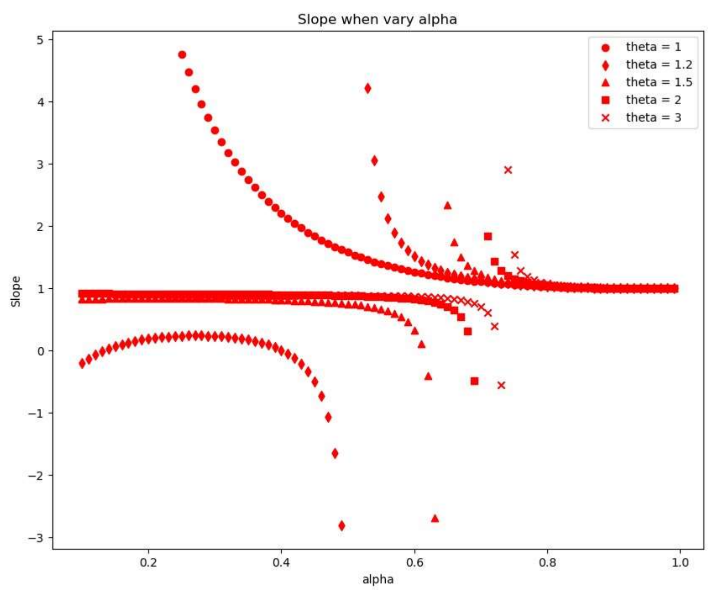
The first-order condition under CRRA utility at equilibrium will be written as

$$(1-\pi)\left[\left((e_i - \bar{k}_i) + A_i\bar{k}_i^{\alpha_i}\right)^{-\theta}\left(1 - A_i\alpha_i\bar{k}_i^{\alpha_i - 1}\right)\right] = \pi\left[A_i\alpha_i\left(A\bar{k}_i^{\alpha}\right)^{-\theta}\left(\bar{k}_i^{\alpha_i - 1}\right)\right]. \tag{14}$$

[4] According to Marquetti (2007), countries with a low labor-capital ratio have an output elasticity to capital (α) of around 0.3–0.5 and 0.6–0.75 for others. In addition, Gandelman’s (2014) commonly accepted measures of coefficient of relative risk aversion (θ) is between 1 and 3.

From the CRRA utility function and Cobb-Douglas production function assumed, we can deduce that the exponential parameters will be the potentially pivoting factors influencing the equilibrium characteristics of the steady state. So, we first start by scanning for the slopes $\left(1 - \frac{MB_1B_2}{B_1N_2+B_2N_1}\right)$ within the standard θ and α values based on past literature. The values of A, N, M, e, and π are arbitrary and only meant for examining the impacts of these parameters on the equilibrium characteristics.

Figure 2. Equilibrium characteristics under varying degrees of risk aversion and output elasticity of capital



3.3 Remarks

Under CRRA utility and Cobb-Douglas production function assumption, sufficient conditions for the existence of a unique solution for a stable equilibrium are: θ with values between 1.2 to 2 AND α with values between 0.1 to 0.45. Several studies, e.g., Layard et al. (2008), Gandelman (2014), and Szpiro (1986), have shown that this range of parameter space, having discussed θ values between 1.2

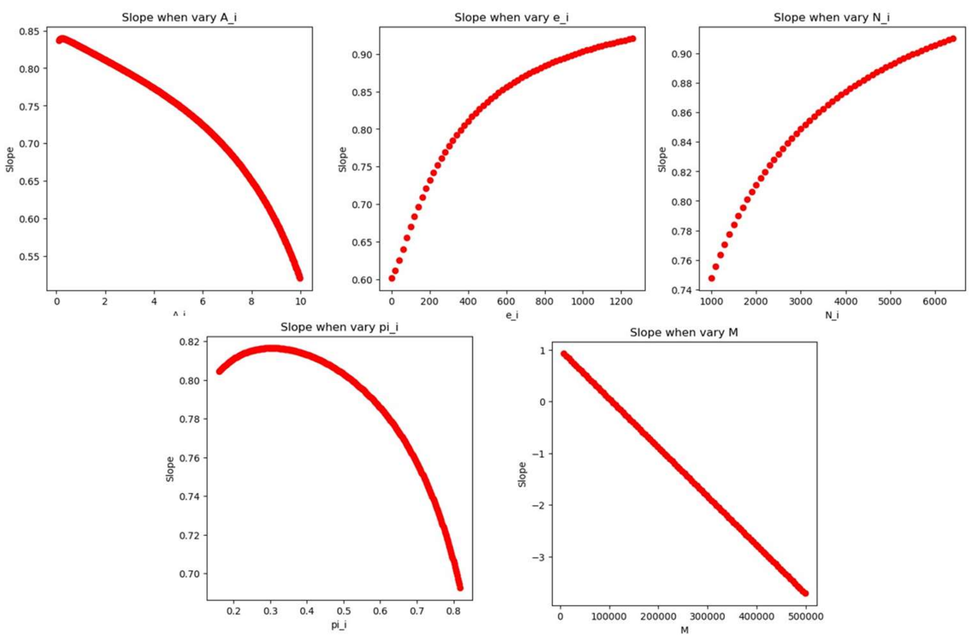
and 2. Harrasova and Roy (2020) show that several countries have α values in the range of 0.1 to 0.45.

From Figure 2, we can see that within the standard range of θ and α, world states exist in which the bubble can exist in a stable equilibrium (slope of equation (11) with values between -1 and 1). When α is lower than a certain threshold, the cryptocurrency bubble exhibits a monotonic stable equilibrium (slope between 0 and 1). Above that threshold, the bubbly equilibrium shifts to an oscillatory stable case (slope between -1 and 0), and an even higher α will now lead to either oscillatory divergent or monotonically divergent equilibria (slope <-1 or >1). Also, note the importance of θ in the characteristics of the equilibrium. When θ is equal to one, the bubbly steady state is never stable. However, when θ is more than one, a low α value can give rise to stable steady states, and as θ increases, the pivoting threshold of α increases.

The intuition behind this finding is that agents choose a certain amount of future consumption under both world states based on their risk aversion level and wealth, along with all the other parameters of the model. When the output elasticity of capital is low, agents need to invest more in capital to achieve their desired amount of consumption in the world state where the bubble bursts. This means that less endowment is being spent on bubbles, allowing the bubble price to be more stable. As α increases, agents can invest less in capital to get the desired level of consumption under the bubbleless world state and invest more in bubbles, causing more volatility. As agents become more risk averse, they will try to smooth consumption between two world states more by investing more in capital or requiring a higher α threshold before shifting more investment to bubbles and causing increased bubble volatility.

We now look at how each parameter impacts steady-state characteristics, assuming θ and α are fixed at the base values in Table 2. Note that due to the nature of the numerical exercise, the results discussed in this section are valid only for the range of parameters specified.

Figure 3. Impact on bubbly equilibrium when varying parameters for A, N, π, M, and e



A and M impact the equilibrium characteristic the same way α does. As A and M increase, the slope of equation (11) decreases, and the equilibrium characteristic can change from monotonic stable to oscillatory stable to oscillatory unstable. As productivity increases, people can invest less in capital to achieve the desired capital-to-bubble ratio at old age with their assumed risk aversion level and invest more in the bubble. This causes the price of the bubble to be less volatile. Similarly, as the supply of the bubble increases, the value of the bubble decreases, allowing more people to participate and leading to less volatile prices. π also exhibits a similar impact, but because π is limited to a max value of 1.0, at the base value of parameters assumed even if π approaches 1.0, it will not change the characteristic of equilibrium. As the risk of bubble bursting increases, the bubble price becomes less volatile. As e and N increase, the slope of equation (11) also increases and approaches 1.0. As people's wealth or the population increases, there is more available purchasing power in the economy to correct the price back to a steady state more rapidly.

Discussion on the volatile nature of cryptocurrency bubbles

In the limiting case where agents have perfect foresight and rational behavior and the equilibrium is found to be unstable, a change in the economy parameters would mean a discrete jump in bubble price from one steady state to the new one. On the other hand, if agents have near-perfect or imperfect foresight, it is possible for the price of the bubble to drift away from the steady state for some length of time until the agents realize that the price is not at a steady state and the price crashes back to the new steady state. When the steady state is oscillatory stable, after a shock in the economy, the bubble price moves between being higher and lower as it approaches the new steady state. When the steady state is monotonic stable but with the slope of equation (11) close to 1.0, the bubble price moves very quickly toward the new steady state following a shock in the economy. All these situations allow us to conclude that cryptocurrency prices are volatile in nature.

4. Cryptocurrency Bubbles and Welfare

We use consumption as the main measure of welfare for agents in the economies with a cryptocurrency bubble. This approach is consistent with the literature on welfare measurement, such as Meyer and Sullivan (2003), who argued that consumption is a better indicator of welfare than income because it reflects permanent income, government programs and credit markets, unofficial or illegal sources of income, and private and government transfers such as donations or benefits.

We use the same model as set up by equations (1) to (8) and follow Thepmonkol (2020) in determining welfare.

Since,

$$c_{2t+1,i,b} = \frac{P_{t+1}}{P_t}\left(e_i - k_{t+1,i}\right) + f\left(k_{t+1,i}\right),$$

taking the derivative of expected consumption yields

$$\frac{\partial E\left(c_{2t+1,i}\right)}{\partial k_{t+1,i}} = (1-\pi_i)\left(-\frac{P_{t+1}}{P_t} + f'\left(k_{t+1,i}\right) + \left(e_i - k_{t+1,i}\right)\frac{\partial \frac{P_{t+1}}{P_t}}{\partial k_{t+1,i}}\right) + \pi_i f'\left(k_{t+1,i}\right) \quad (15)$$

Hence, expected consumption increases in the presence of a bubble when $\frac{\partial \frac{P_{t+1}}{P_t}}{\partial k_{t+1,i}}$ is negative because as investment in capital drops, shifting to a bubble, the expected consumption increases. For this statement to be true, we also require that the expected price ratio or return from bubble is higher than the expected return from investment in capital so that equation (15) becomes negative. To this note, similar to Tirole's (1985) proposition, bubbles crowd out productive investment and, in the process, increase the marginal productivity of capital, which leads to increased welfare.

Proposition 2

For any concave utility function and production function in an economy with world bubble, the return of bubble is more than the marginal return of capital.

Proof

From the first order condition,

$$(1-\pi_i)u'\left(c_{2t+1,i,b}\right)\left(\frac{P_{t+1}}{P_t}-f'^{(k_{t+1,i})}\right)= \ \pi_i u' \ \left(c_{2t+1,i,nb}\right)\left(f'\left(k_{t+1,i}\right)\right).$$

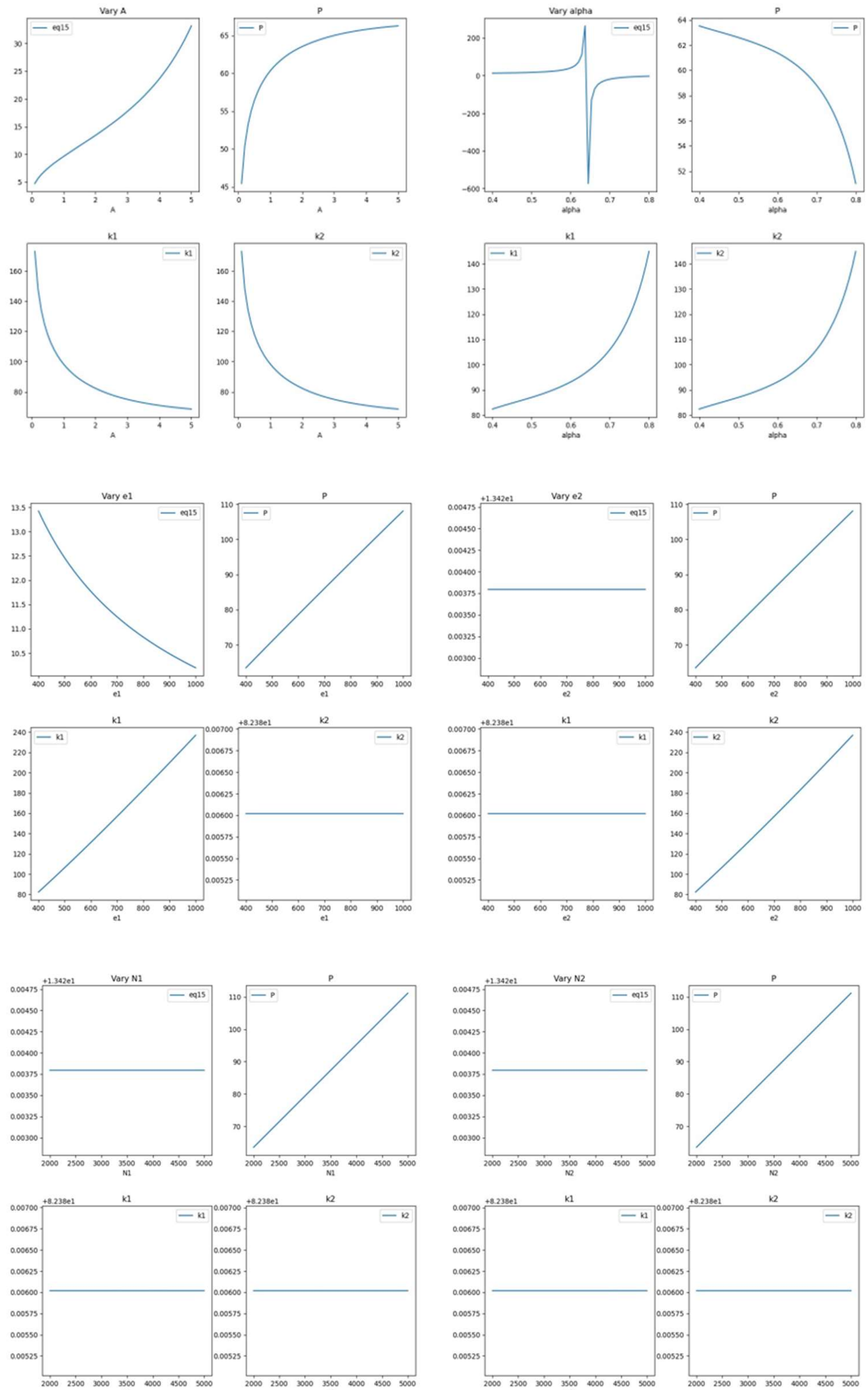
Consumption is lower in the world state where the bubble bursts, so the marginal utility from the consumption in that world state must be higher than the marginal utility from the world state where the bubble persists. For the first-order condition to hold so that both sides of the equation are equal, the expected return from the bubble must be higher than the expected marginal return from capital, $(1-\pi_i)\left(\frac{P_{t+1}}{P_t}-f'\left(k_{t+1,i}\right)\right) > \pi_i\left(f'\left(k_{t+1,i}\right)\right)$. Hence, proposition 2 is proved.

To determine the value of equation (15), we need to first find the value of $\frac{d\frac{P_{t+1}}{P_t}}{dk_{t+1}}$. We determine this by finding the partial derivative of equation (7) with respect to capital $k_{t+1,i}$.

$$\frac{d\frac{P_{t+1}}{P_t}}{dk_{t+1}}=\frac{\frac{\pi}{(1-\pi)}\left[u'(C_2^{1nb})f''(k_{t+1}^1)+f'(k_{t+1}^1)^2u''(C_2^{1nb})\right]+u'(C_2^{1b})f''(k_{t+1}^1)+u''(C_2^{1b})\left(\frac{P_{t+1}}{P_t}-f'(k_{t+1}^1)\right)^2}{u'(C_2^{1b})+\left(\frac{P_{t+1}}{P_t}(e_1^1-k_{t+1}^1)+f(k_{t+1}^1)\right)u''(C_2^{1b})-\left(f'(k_{t+1}^1)(e_1^1-k_{t+1}^1)+f(k_{t+1}^1)\right)u''(C_2^{1b})} \quad (16)$$

From proposition 2, we know that as long as equation (16) is negative, equation (15) will also be negative, and the bubbles will create welfare for agents. Once again, it is not possible to determine analytically whether equation (16) is negative, so we resort to a numerical approach to determine the value of equation (16). The value of equation (16) can then be used to determine the value of equation (15). For simplicity, since we assume the two countries to be identical, we show the graphs for equation (15) for Country 1 only. The results for Country 2 will be symmetrical.

Figure 4. Bubble’s welfare creation when parameters of the world vary



When productivity increases, young agents reduce their capital investment and increase their demand for the bubble, driving up its price. However, equation (15) remains positive for all values of A. This implies that the bubble returns decrease as young agents invest more in the bubble and less in capital. Therefore, the bubble does not improve welfare. When the output elasticity of capital increases, young agents increase their capital investment and reduce their demand for the bubble, driving down its price. However, equation (15) is not always positive for different values of α. There is a threshold value of α above which equation (15) becomes negative. This means that when the output elasticity of capital is high, shifting from capital to the bubble increases welfare. Note that the bubble price is an endogenous variable that depends on all the parameters of the model, including the risk aversion of agents. Therefore, the threshold value of α varies with the level of risk aversion.

To explain why agents react differently concerning capital and bubble investment decisions to changes in A and α, we need to examine more closely the marginal returns that change as these parameters change. The marginal returns of bubble and capital can be described by the following equation:

$$(1-\pi_i)\frac{P_{t+1}}{P_t}\left(\frac{P_{t+1}}{P_t}\left(e_i-k_{t+1,i}\right)+A_i k_{t+1,i}{}^{\alpha_i}\right)^{-\theta} = A_i\alpha k_{t+1,i}{}^{\alpha_i-1}\left[\pi_i\left[\left(A_i k_{t+1,i}{}^{\alpha_i}\right)^{-\theta}\right]+(1-\pi_i)\left(\frac{P_{t+1}}{P_t}\left(e_i-k_{t+1,i}\right)+A_i k_{t+1,i}{}^{\alpha_i}\right)^{-\theta}\right]. \quad (17)$$

The LHS of the equation is the total expected marginal return from bubble, and the RHS of the equation is the total expected marginal return from capital. Agents will invest more in a bubble if the LHS exceeds RHS and vice versa until they equalize again in the steady state. Now, we look at the changes to the returns when A and α increase.

Table 2. Returns from bubble and capital changes with changes in A and α

A	2	2.4
bubble return	0.000151	0.000147
capital return	0.000151	0.000145
α	0.4	0.5
bubble return	0.000151	0.000149
capital return	0.000151	0.000151

We can see that, as expected, A and α should shift both the bubble return and capital returns in the same direction. However, the degree to which the returns of the bubble and the returns of the capital investment change is different and governed by the relative risk aversion coefficient. When A increases, the marginal returns of both bubble and capital decrease, but the marginal return from capital decreases more, so the agents' lower investment in capital and increase investment in bubble as shown. On the other hand, the marginal return from the bubble decreases more than that from the capital when α increases. Hence, agents increase the investment in capital and lower investment in bubble when α increases.

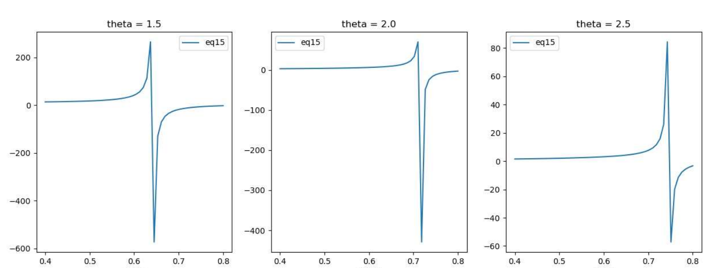
The level of wealth in the two countries changes the level of investment in capital as well as the bubble price. As people become wealthier, they can invest more in both bubble and capital. The ratio between bubble and capital investment is the same, but the absolute amount spent on each increases with higher wealth. However, since the supply of bubble is fixed, as there is more demand for bubble, the price of the bubble also increases. In terms of welfare, as wealth increases, the value of equation (15) approaches zero but remains positive.

Changing the maximum amount of cryptocurrency does not affect the investment choice of young agents between the bubble and capital. Since this change does not alter the model parameters, agents' wealth or preferences, it only dilutes the value of the bubble. Unlike traditional stock splits that may increase the value of stocks by making them cheaper and more liquid, cryptocurrencies are infinitely divisible, and agents can buy any fraction of them. Therefore, the amount of cryptocurrency has no impact on the economy.

When the risk of the bubble bursting in old age is higher, agents invest more in capital and less in the bubble. This lowers the demand and the price of the bubble. Intuitively, higher risk does not improve welfare when agents invest more in the bubble. Finally, the population size of each country does not affect the individual investment in capital and the bubble. The capital investment per person is constant because we assume unlimited capital supply in our model. The bubble price rises because the bubble supply is fixed, but the demand increases with the population.

This section has shown that bubbles improve welfare when the relative risk aversion coefficient is below a certain threshold. This threshold depends on α, the output elasticity of capital. Higher α implies a higher threshold, and vice versa. Similarly, higher risk aversion implies higher α for bubbles to increase welfare, and vice versa.

Figure 5. Relationship between α and θ in determining bubble's welfare creation



5. Contagion Impact of Cryptocurrency

This section examines how cryptocurrency bubbles transmit shocks across countries. We use the model from the previous chapter with both cryptocurrency bubbles and capital. We simplify the analysis by using comparative statistics to show how a shock in one country affects itself and another country that trades the same cryptocurrency bubble. We assume that there are parameters that support a stable bubbly equilibrium, but we leave the analysis of the adjustment dynamics after a shock using impulse response functions to future research. We focus on how cryptocurrency bubbles act as contagion channels affecting steady-state prices and investment decisions across countries. The system is highly nonlinear and complex, so we cannot analytically determine whether a shock in one country has positive or negative effects on another country's investment and consumption decisions. Therefore, we use the Newton numerical method which uses reiterations that produce successively better approximations of the roots to show the contagion features of cryptocurrency bubbles. For all cases shown below, we assume that agents in all countries have θ of 1.5.

5.1 Shock to Capital Productivity

Recall that the system of first-order equations and the market clearing conditions are as follows:

$$(1-\pi_i)\left(\frac{P_{t+1}}{P_t}\left(e_i-k_{t+1,i}\right)+e^{z_i}A_i {k_{t+1,i}}^{a_i}\right)^{-\theta}\left[\frac{P_{t+1}}{P_t}-e^{z_i}A_i\alpha {k_{t+1,i}}^{a_i-1}\right]=$$

$$\pi_i\left[\left(e^{z_i}A_i {k_{t+1,i}}^{a_i}\right)^{-\theta}e^{z_i}A_i\alpha {k_{t+1,i}}^{a_i-1}\right] \quad (18)$$

$$\text{and } N_1\left(e_1-k_{t+1,1}\right)+N_2\left(e_2-k_{t+1,2}\right)=MP_t. \quad (19)$$

We simplify the analysis by assuming first that both countries are identical, with an equal endowment e_i of 400 and the same initial total factor productivity A of 2. Each country has a population of 2000 with no growth, and there is a fixed supply of 20000 units of infinitely divisible bubble. The probability of the bubble bursting π is 0.1, and α is 0.4. z_i is the deterministic exogenous one-time shock to productivity of country i.

Table 3. Bubble price and investment decisions under different productivity shocks

Parameters	e1	e2	Pbar	k1bar	k2bar	z1	z2	C1 B	C1 NB	C2 B	C2 NB
Steady State same wealth	400	400	63.53	82.33	82.33	0	0	329.34	11.68	329.34	11.68
Productivity increase in country 2 same wealth	400	400	63.93	82.33	78.33	0	0.2	329.33	11.68	335.65	13.98
Steady State different wealth	100	700	63.35	17.36	149.2	0	0	88.90	6.26	565.63	14.81
Productivity increase in richer country	100	700	64.16	17.36	141.1	0	0.2	88.87	6.25	576.59	17.69
Productivity increase in poorer country	100	700	63.40	16.85	149.2	0.2	0	90.71	7.561	565.63	14.81

Notes: C1 B refers to the consumption of agents in Country 1 when the bubble persists, and C1 NB refers to the consumption of agents in Country 1 when the bubble bursts, and similarly for Country 2.

When there is a positive shock to productivity in Country 2, the agents in Country 2 reduce their investment in capital and demand more bubbles. This leads to an increase in the price of the bubble. The agents in Country 1, which do not experience the positive shock to productivity, now have to buy bubbles at a higher price so they can afford less capital, causing a slight drop (4^{th} decimal drop) in capital investment. Agents in Country 2 benefit from an increase in consumption in both world states, while agents in Country 1 face a slight drop in consumption in the bubble state due to the higher bubble price and lower investment in capital.

The case when shocks happen to countries with different wealth is like when agents had identical wealth, with the only difference being that the impact of the productivity shock on the bubble price is higher when the shock occurs in the richer country than in the poorer country. The change in investment in capital in Country

2, where the shock happens, is also higher when Country 2 is richer. Moreover, the bigger price change in the bubble due to the shock happening in the richer country also causes a bigger change in the proportion of income to investment between capital and bubble in the poorer country, where no shock has occurred. The insight that we gain here is that with the existence of a world bubble, the same magnitude of shock happening in the richer country will have a much bigger impact in both the country where the shock occurs and the smaller countries without productivity shock, but they are affected indirectly by the bubble price that they also hold. On the other hand, if the productivity shock happens in the poorer country, it will also get a positive boost to consumption in both world states equal to what the rich country receives when the shock occurs in the rich country in terms of percentage, though a much smaller boost in absolute terms.

5.2 Shock to Bubble Risk With Risk Propagation Across Borders

In this section, we will modify our model so that not only do prices propagate bubble risk shocks to other countries, but the risk shock in one country itself propagates directly to the risk perception of agents in other countries:

$$\left(1 - e^{z_i}\pi_i(e^{\tau z_j})\right)\left[\left(\frac{P_{t+1}}{P_t}\left(e_1^1 - k_{t+1,i}\right) + A_i {k_{t+1,i}}^{a_i}\right)^{-\theta}\left[\frac{P_{t+1}}{P_t} - A_i a_i {k_{t+1,i}}^{a_i - 1}\right]\right] =$$
$$e^{z_i}\pi_i(e^{\tau z_j})\left[\left(A_i {k_{t+1,i}}^{a_i}\right)^{-\theta} A_i a_i {k_{t+1,i}}^{a_i - 1}\right]. \tag{20}$$

We have now introduced a direct propagation of bubble risk shock from one country to another with a discount factor of tau (τ) and z_i is the deterministic exogenous one-time shock in country i. Think of tau as the degree of correlation of perceived risk propagation from one country to another. For example, in the real world, if the government of Country 2 imposes a ban on trading cryptocurrencies on exchanges within its borders, agents in Country 1 learn of this immediately and feel that due to the drop in participation of agents in Country 2 in the cryptocurrency market, they also face an increased risk to the persistence of the bubble in the future. However, agents in Country 1 are still able to trade cryptocurrencies on the exchanges within their own country's borders or even on exchanges in other countries that have not banned cryptocurrency trading. Recall that the banning of trading cryptocurrencies within a country's border does not mean that trade of cryptocurrencies within that country drops to zero. Due to the nature of cryptocurrencies, agents in the banned country may still transfer their cryptocurrencies out to exchanges in other countries to trade. However, it will become much more difficult and resource-consuming (fees spent on currency exchanges and transfers in order to trade on exchanges based elsewhere) for agents in the banned country, hence lowering the amount of demand and increasing the chance that the bubble will burst.

We start directly with the case of the two countries having different amounts of wealth. Since the agents in the richer country have more demand and purchasing power, the drop of the bubble price caused by a shock to productivity is much higher in magnitude when compared to the case when the shock happens in the poor country.

Table 4. Bubble price and investment decisions under different risk perception shocks

Parameters	e1	e2	Pbar	k1bar	k2bar	τ	z1	z2	C1 B	C1 NB	C2 B	C2 NB
Steady State different wealth	100	700	63.35	17.36	149.2	0.5	0	0	88.90	6.26	565.63	14.81
Risk perception increase in richer country	100	700	61.13	18.60	170.1	0.5	0	0.2	87.84	6.44	545.53	15.61
Risk perception increase in poorer country	100	700	62.06	19.85	159.6	0.5	0.2	0	86.76	6.61	555.59	15.22
Risk perception increase in poorer country	100	700	61.01	19.85	170.1	1	0.2	0	86.76	6.61	545.53	15.61

Table 4 provides another interesting insight into our examination of responses to bubble risk propagation. In traditional assets like housing or stocks, shocks happening in much poorer countries do not usually lead to significant investment decisions and consumption changes in the richer countries. We know for a fact that with traditional assets like stocks, usually, only smaller stock exchanges follow the movements of the larger stock exchanges. However, when a world bubble like cryptocurrency exists, investment and consumption shocks happening in smaller, less significant markets may also lead to significant changes in investment and consumption choices in larger, more influential markets.

6. Conclusion

We have shown that cryptocurrency has gained popularity and is both an opportunity and a concern for many countries and their governments. From the two most popular cryptocurrencies today, Bitcoin and Ethereum, it is apparent that cryptocurrencies can readily be classified as bubbles because they are traded at astoundingly high prices while not having any underlying rights to physical assets or cash flows. Under the most common assumption of risk-averse agents, suitable output elasticity of capital-intensive economy, and relatively high level of risk from world assets like cryptocurrency, we can conclude that world assets can have an array of equilibrium characteristics. When the relative risk aversion is greater than one, the output elasticity of capital determines whether cryptocurrency bubbles create welfare. The output elasticity of capital must be higher than a certain threshold for bubbles to continue to increase welfare for agents. As the relative risk aversion of agents increases, this threshold increases. The world bubble also plays a direct role in transferring shocks from one country to another, as shown in our contagion models. A shock in the productivity of capital in one country is transferred to another country through the change in price of the world bubble. Unlike traditional country-restricted bubbles, bubble risk changes in one country can easily transmit to another country. A direct transfer of bubble risk leads to a stronger impact on the world bubble price and the reaction of the agents in the country not affected by the original shock. A very interesting implication is that a shock to bubble risk in a small economy can lead to a very big change in risk perception and investment decision changes in a much larger and richer economy. The insights from this paper show us that cryptocurrency bubbles have the potential to create more welfare and make agents better off. World bubbles may help increase consumption for countries with lower capital output since their prices are also determined by other better-off countries.

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Appendix I

Derivation of equation (11):

From FOC (7)

$$(1-\pi_i)u'(c_{2t+1,i,b})\left(\frac{P_{t+1}}{P_t}-f'(k_{t+1,i})\right)=\pi_i u'(c_{2t+1,i,nb})\left(f'(k_{t+1,i})\right)$$

Linearize

$$\begin{aligned}&\left[(1-\pi_i)u'(c_{2t+1,i,b})\left(\frac{1}{\bar{P}}\right)+\left(1-f'(k_{t+1,i})\right)(1-\pi_i)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\left(\frac{1}{\bar{P}}\right)\right](P_{t+1}-\bar{P})\\&+\left[(1-\pi_i)u'(c_{2t+1,i,b})\left(-\frac{1}{\bar{P}}\right)+\left(1-f'(k_{t+1,i})\right)(1-\pi_i)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\left(-\frac{1}{\bar{P}}\right)\right](P_t-\bar{P})\\&+\left[(1-\pi_i)u'(c_{2t+1,i,b})\left(-f''(k_{t+1,i})\right)+\left(1-f'(k_{t+1,i})\right)(1-\pi_i)u''(c_{2t+1,i,b})\left(-1+f'(k_{t+1,i})\right)\right](k_{t+1,i}-\bar{k}_i)\\&=\left[\pi_i u'(c_{2t+1,i,nb})\left(f''(k_{t+1,i})\right)+\left(f'(k_{t+1,i})\right)\pi_i u''(c_{2t+1,i,nb})f'(k_{t+1,i})\right](k_{t+1,i}-\bar{k}_i)\end{aligned}$$

$$\begin{aligned}&\left[(1-\pi_i)u'(c_{2t+1,i,b})\left(\frac{1}{\bar{P}}\right)+\left(1-f'(k_{t+1,i})\right)(1-\pi_i)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\left(\frac{1}{\bar{P}}\right)\right](P_{t+1}-\bar{P})\\&=-\left[(1-\pi_i)u'(c_{2t+1,i,b})\left(-\frac{1}{\bar{P}}\right)+\left(1-f'(k_{t+1,i})\right)(1-\pi_i)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\left(-\frac{1}{\bar{P}}\right)\right](P_t-\bar{P})\\&+\left[\pi_i u'(c_{2t+1,i,nb})\left(f''(k_{t+1,i})\right)+\left(f'(k_{t+1,i})\right)\pi_i u''(c_{2t+1,i,nb})f'(k_{t+1,i})\right.\\&\left.-\left((1-\pi_i)u'(c_{2t+1,i,b})\left(-f''(k_{t+1,i})\right)+\left(1-f'(k_{t+1,i})\right)(1-\pi_i)u''(c_{2t+1,i,b})\left(-1+f'(k_{t+1,i})\right)\right)\right](k_{t+1,i}-\bar{k}_i)\end{aligned}$$

$$\begin{aligned}&\frac{(1-\pi_i)}{\bar{P}}\left[u'(c_{2t+1,i,b})+\left(1-f'(k_{t+1,i})\right)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\right](P_{t+1}-\bar{P})\\&=\frac{(1-\pi_i)}{\bar{P}}\left[u'(c_{2t+1,i,b})+\left(1-f'(k_{t+1,i})\right)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\right](P_t-\bar{P})\\&+\left[\pi_i\left(u'(c_{2t+1,i,nb})\left(f''(k_{t+1,i})\right)+\left(f'(k_{t+1,i})\right)^2u''(c_{2t+1,i,nb})\right)\right.\\&\left.+(1-\pi_i)\left(u'(c_{2t+1,i,b})\left(f''(k_{t+1,i})\right)+\left(1-f'(k_{t+1,i})\right)^2u''(c_{2t+1,i,b})\right)\right](k_{t+1,i}-\bar{k}_i)\end{aligned}$$

$$\begin{aligned}&\left[\pi_i\left(u'(c_{2t+1,i,nb})\left(f''(k_{t+1,i})\right)+\left(f'(k_{t+1,i})\right)^2u''(c_{2t+1,i,nb})\right)\right.\\&\left.+(1-\pi_i)\left(u'(c_{2t+1,i,b})\left(f''(k_{t+1,i})\right)+\left(1-f'(k_{t+1,i})\right)^2u''(c_{2t+1,i,b})\right)\right](k_{t+1,i}-\bar{k}_i)\\&=\frac{(1-\pi_i)}{\bar{P}}\left[u'(c_{2t+1,i,b})+\left(1-f'(k_{t+1,i})\right)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\right][(P_{t+1}-\bar{P})-(P_t-\bar{P})]\end{aligned}$$

$$[(P_{t+1}-\bar{P})-(P_t-\bar{P})]=\frac{\left[\pi_i\left(u'(c_{2t+1,i,nb})\left(f''(k_{t+1,i})\right)+\left(f'(k_{t+1,i})\right)^2u''(c_{2t+1,i,nb})\right)+(1-\pi_i)\left(u'(c_{2t+1,i,b})\left(f''(k_{t+1,i})\right)+\left(1-f'(k_{t+1,i})\right)^2u''(c_{2t+1,i,b})\right)\right]}{\frac{(1-\pi_i)}{\bar{P}}\left[u'(c_{2t+1,i,b})+\left(1-f'(k_{t+1,i})\right)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\right]}(k_{t+1,i}-\bar{k}_i)$$

$$B_i=\frac{\left[\pi_i\left(u'(c_{2t+1,i,nb})\left(f''(k_{t+1,i})\right)+\left(f'(k_{t+1,i})\right)^2u''(c_{2t+1,i,nb})\right)+(1-\pi_i)\left(u'(c_{2t+1,i,b})\left(f''(k_{t+1,i})\right)+\left(1-f'(k_{t+1,i})\right)^2u''(c_{2t+1,i,b})\right)\right]}{\frac{(1-\pi_i)}{\bar{P}}\left[u'(c_{2t+1,i,b})+\left(1-f'(k_{t+1,i})\right)u''(c_{2t+1,i,b})(e_i-k_{t+1,i})\right]}$$

$$[(P_{t+1}-\bar{P})-(P_t-\bar{P})]=B_i(k_{t+1,i}-\bar{k}_i)$$

$$\left(k_{t+1,i} - \bar{k}_i\right) = \frac{(P_{t+1} - \bar{P}) - (P_t - \bar{P})}{B_i}$$

$$\left(k_{t+1,1} - \bar{k}_i\right) = \frac{(P_{t+1} - \bar{P}) - (P_t - \bar{P})}{B_1}$$

$$\left(k_{t+1,2} - \bar{k}_i\right) = \frac{(P_{t+1} - \bar{P}) - (P_t - \bar{P})}{B_2}$$

$$N_1 m_{t,1} + N_2 m_{t,2} = M$$

$$e_i = P_t m_{t,i} + k_{t+1,i}$$

$$m_{t,i} = \frac{\left(e_i - k_{t+1,i}\right)}{P}$$

$$N_1\left(\frac{\left(e_1 - k_{t+1,1}\right)}{P}\right) + N_2\left(\frac{\left(e_2 - k_{t+1,2}\right)}{P}\right) = M$$

$$N_1\left(e_1 - k_{t+1,1}\right) + N_2\left(e_2 - k_{t+1,2}\right) = MP$$

$$-N_1\left(k_{t+1,1} - \bar{k}_1\right) - N_2\left(k_{t+1,2} - \bar{k}_2\right) = M(P_t - \bar{P})$$

$$\left(k_{t+1,1} - \bar{k}_1\right) = -\frac{M}{N_1}(P_t - \bar{P}) - \frac{N_2}{N_1}\left(k_{t+1,2} - \bar{k}_2\right)$$

$$(P_{t+1} - \bar{P}) = (P_t - \bar{P}) + B_1\left(k_{t+1,1} - \bar{k}_i\right)$$

$$(P_{t+1} - \bar{P}) = (P_t - \bar{P}) + B_1\left(-\frac{M}{N_1}(P_t - \bar{P}) - \frac{N_2}{N_1}\frac{(P_{t+1} - \bar{P}) - (P_t - \bar{P})}{B_2}\right)$$

$$(P_{t+1} - \bar{P}) = (P_t - \bar{P}) - B_1\frac{M}{N_1}(P_t - \bar{P}) - \frac{N_2}{N_1}\frac{B_1}{B_2}(P_{t+1} - \bar{P}) + \frac{N_2}{N_1}\frac{B_1}{B_2}(P_t - \bar{P})$$

$$\left(1 + \frac{N_2}{N_1}\frac{B_1}{B_2}\right)(P_{t+1} - \bar{P}) = \left[1 - B_1\frac{M}{N_1} + \frac{N_2}{N_1}\frac{B_1}{B_2}\right](P_t - \bar{P})$$

$$\frac{N_1 B_2 + N_2 B_1}{N_1 B_2}(P_{t+1} - \bar{P}) = \frac{N_1 B_2 - M B_1 B_2 + N_2 B_1}{N_1 B_2}(P_t - \bar{P})$$

$$(P_{t+1} - \bar{P}) = \frac{N_1 B_2 + N_2 B_1 - M B_1 B_2}{N_1 B_2 + N_2 B_1}(P_t - \bar{P})$$

$$(P_{t+1} - \bar{P}) = \left(1 - \frac{M B_1 B_2}{N_1 B_2 + N_2 B_1}\right)(P_t - \bar{P})$$