Southeast Asian Journal of Economics 2(1), June 2014: 23-50

Asset Pricing Puzzle in Emerging Markets: Evidence from Thailand

Somphop Duangthong
School of Economics, University of Thai Chamber of Commerce, Bangkok, Thailand
Corresponding author: somphop@panrcgroup.com

Abstract

The feasible region of mean-standard deviations of stochastic discount factors, called the Hansen Jagannathan bound, is constructed using quarterly real returns on Thailand's stocks and savings during 1993-2010. The stock returns are calculated stock by stock, including capital gains, dividends, and values of rights offering. The asset pricing puzzle is detected. The observed volatility of stochastic discount factors, generated by the standard Real Business Cycle model, is inconsistent with the feasible region. The inclusion of the 1997-8 Asian financial crisis alleviates the puzzle, but inadequately. I use alternative stochastic discount factors generated by different utility functions. Habit persistence structure increases the volatility of discount factors and partly resolves the puzzle. However, the puzzle still remains.

Keywords: Asset Pricing Puzzle, Hansen Jagannathan Bound, Total Return Index, Asian Financial Crisis, Habit Persistence Utility

1. Introduction

The consumption-based capital asset pricing model (C-CAPM) provides a classical description of a representative's behavior regarding asset returns. It illustrates both cross-sectional and intertemporal relationship between consumption and asset payoffs. Although it has been influential in financial economics for a long time, it apparently fails in empirical calibration. The Euler equations implied by the model are consistently rejected, e.g. Grossman and Shiller (1981), Hansen and Singleton (1982, 1983), Grossman, Melino, and Shiller (1985), and Tesar and Werner (1994). Meanwhile, Mehra and Prescott (1985) demonstrate that the historical U.S. stocks' risk premium (the U.S. stock returns in excess of the U.S. T-bill rate) is so large that it cannot be explained by the model, called *"the equity premium puzzle."*

Mehra and Prescott employ a variant of Lucas-Breeden pure exchange economy, using the time- and state-separability of the representative agent's utility. They find that the preference parameters that match the observed equity premium are unreasonable. The representative investor has to be implausibly high risk averse, i.e. the coefficient of risk aversion is to be greater than ten. The puzzle is very robust (see Kocherlakota 1990 and Mehra 2003), based on only three assumptions; which are, the standard constant relative risk aversion (CRRA) utility of a representative agent, a *complete* and *frictionless* asset market. Many studies calibrate the puzzle repeatedly, but they cannot resolve the puzzle yet.

An alternative calibration approach is based on the stochastic discount factors, which quantitatively relate the asset price with its expected future payoffs (see Hansen and Richard 1987). Hansen and Jagannathan (1991) use asset payoffs data, rely on such relationship, and then characterize a feasible region of volatility of stochastic discount factors called the Hansen Jagannathan bound. The bound maps the minimum standard deviation of stochastic discount factors for any given mean of discount factors. Its major advantage is that it is valid for any number of assets and for without riskfree rate.

Another advantage comes from the property of the bound. The additional assets used in constructing the bound always narrow the feasible region, or more restrict the volatility of stochastic discount factors. Hence it avoids the *survivorship bias*. Since I can collect the asset data only on the existing stocks in the market (no data of the delisted stocks), I often overestimate the stock returns. The calibration of the equity premium puzzle is then problematic because the observed equity premium is biased towards a large value. In contrast, the detection of the puzzle using HJ bound is still robust even without the delisted stocks.

"Asset pricing puzzle" is detected as that the volatility of stochastic discount factors generated by the C-CAPM using consumption data do not coincide the feasible region with reasonable parameters. The observed volatility of stochastic discount factors coincides with the feasible values generated by the HJ bound only with an unacceptably high risk aversion or with a subjective discount factor much greater than one. The specification of utility function, which gives different formulas of the stochastic discount factors, is then critical. This study also calibrates the asset pricing puzzle with alternative utility functions.

The puzzle motivates many researchers to explain its causes. These include both measurement bias and theoretical model adjustment. About the measurement, the attempts are to uncover data that would make the asset returns riskier or expected returns smaller; for example, a small chance of an economic catastrophe that is worried by investors (Reitz, 1988 and Barro, 2006), an observably small standard deviation of the U.S. stock returns due to its mean reversion process (Poterba and Summers, 1988), and a distortion due to survivorship bias. Blanchard (1993) and Fama and French (2001) estimates the expected rate of asset returns, rather than a use of realization. They suggest that investors expect lower rate of returns than the realization.

As commented in Reitz (1988), an investor actually considers a small chance of an economic catastrophe but may not really happen. Although an investor's expectation about the crisis should be included in the measurement of risk, it may not been observed in the study of asset pricing puzzle using the realization of stock returns. To study the asset pricing puzzle for Thailand stock market has one major advantage. An economic catastrophe really happened, that is the Asian financial crisis in 1997-8. The stock market was heavily hit. A huge loss and risk of stock returns are observed. The evidence shows that the market index had dropped from the highest point at 1,754 in January 1994 to only 207 in September 1998. An investor had lost their money almost 90% in a few years.

The main modification on the theoretical model is alternative utility functions, including the Epstein-Zin-Weil Generalized Expected Utility (see Epstein and Zin 1989 and Weil 1989), habit persistence utility (see Abel 1990, Constantinides 1990, Gali 1994, and Campbell and Cochrane 1999), and myopic loss aversion (Benartzi and Thaler 1995). The Epstein-Zin-Weil GEU separates the cross-sectional and intertemporal behaviors. It is characterized by a constant coefficient of relative risk aversion and a constant but distinct elasticity of intertemporal substitution. Weil (1989) finds another anomaly called *"the riskfree rate puzzle."* It is the case that although the Treasury bill offers only a low rate of return, individuals still save enough to generate their high rate of consumption growth.

For habit persistence utility, it poses that people are accustomed to their own standard of living. The habit formation assumes the utility depends not only on the current consumption, but also the past levels. The utility is then not time separable, but exhibits habit persistence. This formation succeeds in modeling an investor to be more averse to consumption volatility. A small drop in consumption generates a large drop in the marginal utility that makes it possible to match the observed assets' price and return with low risk aversion. The literature involves two distinctive specifications: *internal* and *external* habit formation. The latter is often known as "keeping up with the Joneses."

Campbell (1999) uses data of 11 developed countries, which are Australia, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, the United Kingdom, and the United States, and finds evidences of the premium puzzle. However, the studies for developing countries are rare. This paper calibrates the asset pricing puzzle for a case of Thailand's asset market. Over 30 years of the Thailand's stock market (SET), the index is highly fluctuated. The stocks seem to be far riskier than those of developed countries. In addition, the consumption growths in developing countries are more volatile.

Sedthapinun (2000) calibrates the Thailand stocks' premium by the same methodology as Mehra and Prescott. Although he concludes that there is no premium puzzle, I retest it for some adjustments. First, this study relaxes the riskfree asset assumption and uses the HJ bound for calibration. Secondly, the stock returns are calculated firm by firm, including share values, dividends, and values of rights offering. Then I constructed 21 portfolios of different SET business categories. This study uses different consumption indicator, which is the national private consumption expenditure in non-durable goods and services developed by the Office of the National Economic and Social Development Board (NESDB). Lastly, this paper calibrates the asset pricing puzzle with alternative utility functions including the GEU and the habit persistence utility.

The rest of the paper is organized as follows. Section 2 presents a description of the data used for analysis including the calculation method of the stock returns. Section 3 describes the theoretical framework in the context of a representative agent C-CAPM with rational expectation. Then Section 4 shows calibration of asset pricing puzzle based on the standard CRRA utility. Robustness check in Section 5 is conducted by using different consumption indicator and by using different asset portfolio. Section 6 compares the results with and without states of the 1997 Asian financial crisis to analyze the impact of the crisis on the asset pricing puzzle. In Section 7, I re-calibrate the puzzle based on alternative stochastic discount factors generated by the GEU and habit persistence utility. Section 8 concludes the paper.

2. Data and Calculation

Data is collected quarterly from Quarter 1, 1993 to Quarter 4, 2010. The period is dictated by the availability of quarterly consumption indicator. This study considers an investment in the stock market as 21 business categories[2] (90 firms) and analyzes only the stocks that have traded since 1993. This study excludes firms that had not been traded more than a quarter, i.e.

---

[2] Agribusiness, Food and Beverage, Fashion, Home & Office Products, Personal Products & Pharmaceuticals, Banking, Finance and Securities, Insurance, Automotive, Packaging, Petrochemicals & Chemicals, Construction Materials, Property Development, Energy & Utilities, Mining, Commerce, Health Care Services, Media & Publishing, Tourism & Leisure, Transportation & Logistics, and Electronic Components

stock suspension, since there is no price data representing the share value in the quarter. This study also excludes firms that had issued warrant during 1993-2010 because of unavailability of warrant's exercise price data.

Data set consists of three series, described as follows:

1. **An investment in stock market** is obtained from the Stock Exchange of Thailand (SET), which consists of:

   1.1 **Real stock price series $\{p_t\}$** is equal to the closed SET price index divided by seasonally-adjusted general consumer price index, developed by Ministry of Commerce (MOC).

   1.2 **Real payoff from dividends in cash $\{d_t\}$** is deflated cash dividend per share.

   1.3 **Real payoff from right in purchasing common shares $\{o_t\}$** is the deflated non-negative surplus of the stock's closed price on the X-date over the exercise price of right (see the calculation). It is assumed non-negativity since the right is optional and a rational investor will exercise it only when it yields a positive payoff.

   1.4 **Movement of shares holding by investors $\{n_t\}$** is the number of common shares proportionate to prior holding shares changed during the period. It includes par changes (i.e. stock splits or reverse splits) and dividends in common shares, but not the stock purchase.

2. **Interest rate of savings, $\{i_t\}$**, makes use of deflated 3-month time deposits average rate of the Thai Commercial Banks[3], developed by Bank of Thailand (BOT). Noted that data of governmental Treasury bills is available only since quarter 1, 2001. Based on continuously compound interest rate, quarterly nominal payoff is calculated as[4] $x_t = e^{i_t/4}$, where i is net interest rate published in

[3] Bangkok Bank, KrungThai Bank, The Siam Commercial Bank, Kasikorn Bank and Bank of Ayudhya

[4] Following the continuously compound interest rate, $x_T = x_0 \cdot e^{iT}$, where i is annual net interest rate and T is year. Then quarterly interest rate $R_1 = x_0 \cdot e^{iT} = 1 \cdot e^{i \cdot 1/4} = e^{i/4}$.

yearly basis and *t* represents quarter. The payoff is then deflated into the real term. Its mean and standard deviation are presented in Table 1.

3. **Real per capita consumption series $\{c_t\}$** makes use of the seasonally adjusted, deflated national private consumption expenditure (PCE) in domestic non-durable goods and services, developed by Office of the National Economic and Social Development Board (NESDB). It is divided by population[5] to represent a representative agent's consumption. Its mean and standard deviation are 0.75% and 0.0191 respectively on quarterly basis, as in Table 1.

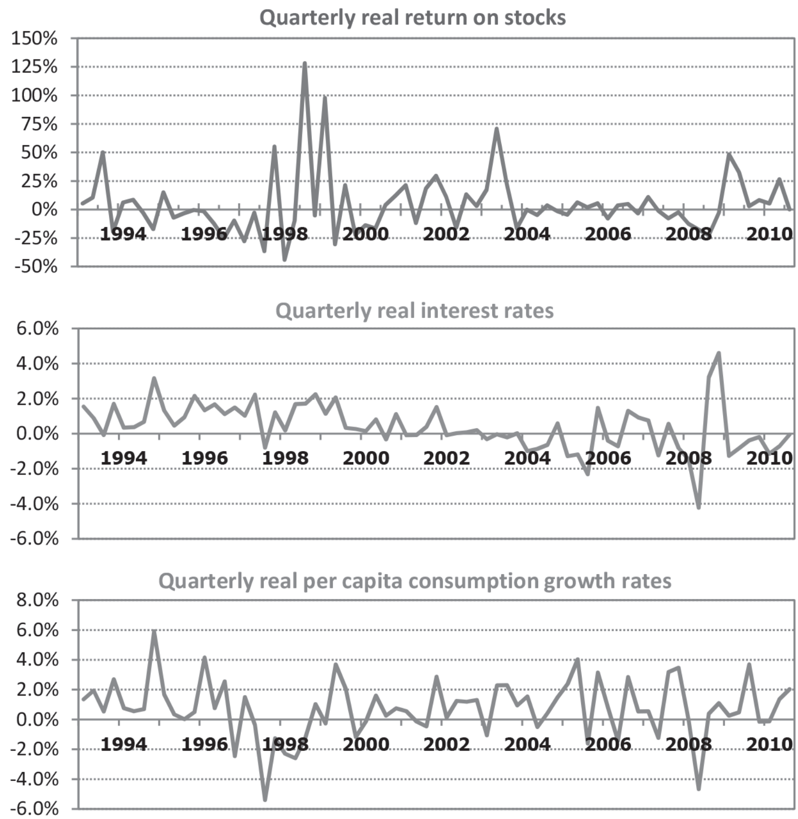
Table 1. Relevant Quarterly Statistics of Thailand's Economy

	Population growth rate	Inflation rate	Real interest rate of savings	Real per capita consumption growth rate
Mean	0.22%	0.86%	0.40%	0.75%
Standard deviation	0.0022	0.0126	0.0134	0.0191

Figure 1 depicts movements of quarterly real returns on stocks, real interest rates, and real per capita consumption growth rates during 1993-2010. The period of study encounters the 1997 Asian financial crisis, by which the consumption levels dropped sharply. The stocks performed badly from 1996-7, but they however recovered during 1998-9. A large consumption drop has occurred again on quarter 3 in 2008 due to an exceptionally high inflation rate, which is 5.11% a quarter.

[5] Before 2002, population is yearly at year end. Quarterly data is computed by constant rate of interpolation.

Figure 1. Quarterly real returns on stocks, real interest rates, and real per capita consumption growth rates, Q1-1993 to Q4-2010



For a long term investment, i.e. from 1993 to 2010, stocks have higher rate of returns and higher risk than the interest rates. Based on quarterly basis, stock's real returns range approximately from -40% to 120%, while real interest rates range only between -4% and 4.5%.

Calculation of the Stock's Returns and Payoffs

To construct the HJ bound, it requires a volatility of asset payoffs in different states of the world. I assume different periods (quarter to quarter) as different states. Then the stock's rate return and a payoff per share at time *t* are calculated as follows

$$R_{i,t} = [(1 + n_{i,t})p_{i,t} + d_{i,t} + o_{i,t}]/p_{i,t-1}, \text{ and } x_{i,t} = p_{i,0}R_{i,t} \quad (1)$$

where $p_{i,t}$ is the closed price of stock i at time t, $d_{i,t}$ is dividend in cash paid to investors who hold stock i from time $t - 1$ to t, $o_{i,t}$ is the benefit from an (optional) right offered to stock-i holders during time t to purchase its new shares at a given price, and $n_{i,t}$ is number of shares that stock-i holders gain during time t per share of time $t - 1$. $1 + n_{i,t}$ represents the gross number of shares gained during period t. It includes the share gain by stock splitting (or loss by reverse splitting) and dividend in stocks. The three forms of payoff can be described as follows:

1. **Capital gain**. $(1 + n_{i,t})p_{i,t}$

In considering capital gain, an investor gains benefit (or loss) from an increase (or a decrease) in the firm's value, mostly reflected by the product of its stock price and shares. However the firm may raise its capital externally, i.e. private placement (PP) and public offering (PO), which do not stand for the return on investment. In addition, the measure of return can be distorted by changing number of shares due to the stock repurchase and the exercise of rights in purchasing stocks.

In other way, I directly calculate the capital gain for investors who hold stock from period to period. The value of shares is the product of the strike price and adjusted number of shares. Investors may automatically obtain new shares as a proportion $n_{i,t}$ to number of prior shares, i.e. from the stock splitting process or dividend in stock. Hence the gross capital gain per share from period $t - 1$ to t is $(1 + n_{i,t})p_{i,t}$. The $n_{i,t}$ is negative in case of reverse splitting.

For example of stock splitting, a firm changes par from 10 to 5 Baht, and splits stock into 2 times. The investors automatically have a double of their shares. Then $1 + n_{i,t}$ is equal to 2.

For example of dividend in stock, a firm decides to pay stock dividend by 4:1 (pay one share of common stock for every four shares). Then $n_{i,t}$ is equal to 1/4, and $1 + n_{i,t}$ is equal to 5/4.

2. **Dividend in cash**. d_t

In practice, the dividend is paid in two types – dividend in cash and dividend in common stock. This type of payoff excludes dividend in stock. Although the dividend in stock gives an investor more shares, it simultaneously dilutes the firm's value through decreasing price. While the firm's value is unchanged, the investors gain no payoff from dividend in stock. As shown above, I already involve the dividend in stock in calculation of capital gain.

The dividend in cash is committed to shares right before the X-date. Changing of share ownership on the X-date or later does not alter the dividend receiver. Capital gain is usually negative since the cash dividend is committed to be paid out. I can observe data that stock prices reflect the dividend activity immediately on the X-date. The payoff is hence assumed to incur on the X-date. This study does not incorporate the discount value of dividend onto the X-date, in the case that the X-date and the payment date are in different quarters, to avoid complexity but insignificantly different.

3. **Right offering to purchase common stocks** (o_t)

The offered right to purchase common shares has two distinct forms – right to purchase common shares and warrant. For convenience purpose, this study will refer the former as "right in common shares." It again employs the X-date principle. The right is offered to shares represented right before the X-date. A change of share ownership on the X-date or later does not alter the right holder. The payoff is again assumed to incur on the X-date.

The "right in common shares" is an optional right for investors who hold common shares right before the X-date and are capable of purchasing newly issued common shares at an agreed price (the exercise price), on a specific date, and in a specific proportion to number of prior shares. The rational investors will exercise the right only when the exercise price is below the strike price on the exercise date, and the payoff is then non-negative.

This study assumes a payoff equal to a non-negative excess of the closed price on the X-date over the exercise price. Denoting $p_{i,t}^{XR}$ the exercise price and $\eta_{i,t}^{XR}$ the exercise proportion to prior shares, the payoff per share from "right in common shares" is calculated as follows:

$$o_{i,t} = \eta_{i,t}^{XR} \cdot \max(p_{i,t} - p_{i,t}^{XR}, 0), \tag{2}$$

where t is denoted exclusively here the X-date.

“Warrant” is quite similar to the “right in common shares”, but its exercise period is extended to several months or years. Warrant holders are periodically able to exercise it during validity period. A rational investor will exercise the warrant when they think it yields the highest (positive) payoff. Unfortunately, the exercise prices of warrant are hardly observed, and hence the calculation of payoff is impossible. Since the validity period is very long, there is much complexity of payoff calculation for warrant.

Table 2 decomposes all three components of stock returns as discussed above. The average stock’s real return is 4.82% per quarter or approximately 20.7% per annum. Capital gain is a majority, but with the highest volatility (risky). Payoffs from dividend in cash and rights in common shares are also significant but with very low risk.

Table 2. Decomposition of quarterly real returns, Q1-1993 to Q4-2010

	Total Net Returns	**Capital gain**	**Dividend in Cash**	**Rights in Common Shares**
Mean	4.82%	3.87%	0.72%	0.22%
Standard deviation	0.2744	0.2734	0.0054	0.0041

Traditionally, the market return is computed annually by considering only market price index and dividend yield[6]. It ignores the benefit from offering rights, so it underestimates the payoff. The estimated variance of returns is however dubious. Although the SET recently publishes total return index (TRI) which includes benefit from rights, the data period is too short for this study. It starts at the beginning of year 2002. In addition, TRI uses a realization of returns. I find that returns from rights in common shares are sometimes negative.

[6] The market’s dividend yield (%) is published in annual rate, which is equal to the market’s total dividend paid out during the latest 12 months relative to the market price index.

Table 3. Quarterly real net returns using three data sources, Q1-2002 to Q4-2010

	Our Study	**SET TRI**	**SET Index & DY (no right)**
Expected market real returns, $E(\mathbf{r}_m)$	5.77%	5.47%	5.34%
Standard deviation, $\sigma(\mathbf{r}_m)$	0.1847	0.1436	N.A.

Table 3 compares the estimated mean rate of stock returns used in this study with market returns, including the traditional one and the SET TRI. It shows quarterly real rate of returns from 2002-2010. Since the market dividend yield is published only in yearly basis, the quarterly rate of return is annualized while the variance is unavailable. As expected, this study gives the highest estimation of returns, while the traditional estimation shows the lowest.

3. Theoretical Framework

I follow the basic pricing equation $\mathbf{p(x)} = E(\mathbf{xm}|\mathbf{I})$ where $\mathbf{p}$ is a current price vector of payoffs $\mathbf{x} = [\mathbf{x}_1', \mathbf{x}_2', \ldots, \mathbf{x}_N']'$ with a vector of their discount factors $\mathbf{m}$, subject to the information set available at current period $\mathbf{I}$. With a complete data set of all assets' payoffs, the stochastic discount factors can be precisely identified. However, the availability of asset data is limited. As well, the agents may practically consider only a limited number of assets. This study relies on such limitation of asset data and generates a feasible mean-standard deviation pairs of $\mathbf{m}$.

I assume the common information sets so that it is identical across agents. The law of iterated expectations is applied to make use of unconditional moment, which easier to estimate the bound than conditional moments. The law of one price is also applied to get a unique $\mathbf{m}$. Hence, the pricing equation results in the following condition:

$$\mathbf{p(x)} = E(\mathbf{xm}). \tag{3}$$

$\boldsymbol{\Sigma} \equiv E[(\mathbf{x} - E(\mathbf{x}))(\mathbf{x} - E(\mathbf{x}))']$ denotes a covariance of the payoffs. If I postulate a discount factor is a linear function of the shocks to the payoffs, then

$$\mathbf{m} = E(\mathbf{m}) + (\mathbf{x} - E(\mathbf{x}))'\mathbf{b} + \varepsilon. \tag{4}$$

I assume $E(\varepsilon) = 0$ and $E(\mathbf{x}\varepsilon) = \mathbf{0}$, and then

$$\mathbf{p} - E(\mathbf{x})E(\mathbf{m}) = \boldsymbol{\Sigma b}, \tag{5}$$

If $\boldsymbol{\Sigma}$ is nonsingular, $\boldsymbol{b} = \boldsymbol{\Sigma}^{-1}[\mathbf{p} - E(\mathbf{x})E(\mathbf{m})]$. Since $E(\varepsilon'\varepsilon) \geq 0$, then

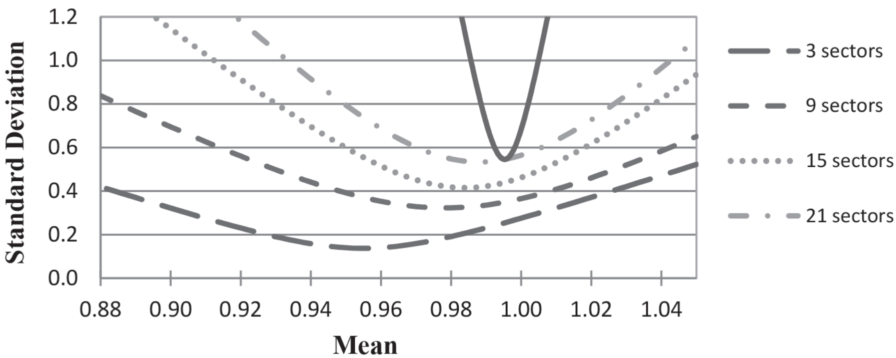
$$\sigma(\mathbf{m}) \geq \{[\mathbf{p} - E(\mathbf{x})E(\mathbf{m})]'\boldsymbol{\Sigma}^{-1}[\mathbf{p} - E(\mathbf{x})E(\mathbf{m})]\}^{1/2}. \tag{6}$$

The mean of the stochastic discount factors determine the feasible values of its standard deviation. It characterizes a hyperbolic region of feasible $\{E(m), \sigma(m)\}$, called Hansen-Jagannathan bound. The value of $E(m)$ determines the minimum value or a lower bound of $\sigma(m)$. And all discount factors must lie in the region. It is constructed by the data about the assets only.

The available stock data excludes stocks delisted from the market. From 1993-2010, fifty three companies were delisted because they failed to achieve satisfactory progress on its rehabilitation. Thirty four companies were delisted among the 1997-8 crisis, and other thirteen companies were under rehabilitation and delisted within two years after that. Literature then questions the calibration in an aspect of *survivorship bias* towards an observed high rate of returns. However, one major property of the HJ bound is that the inclusion of additional asset(s) will always sharpen the bound (narrower), or it more restricts the volatility of stochastic discount factors. It hence avoids the survivorship bias. The asset pricing puzzle and the equity premium puzzle will be reinforced as including the delisted stocks or other assets.

Figure 2 shows an example of adding assets in construct the HJ bound. I find that the feasible region (upper region of the bound) is always narrower. The relatively riskfree rate implies an inverse of $E(\mathbf{m})$. When I add the relatively riskfree asset, like savings, the bound becomes much sharper.

Figure 2. Property of Hansen Jagannathan bound: An example of adding assets



"3 sectors" includes Agribusiness, Automotive, and Banking
"9 sectors" adds Commerce, Construction Materials, Electronic Components, Energy & Utilities, Fashion, and Finance and Securities
"15 sectors" adds Food and Beverage, Health Care Services, Home & Office Products, Insurance, Media & Publishing, and Mining
"21 sectors" adds Packaging, Personal Products & Pharmaceuticals, Petrochemicals & Chemicals, Property Development, Tourism & Leisure, and Transportation & Logistics
"21 sectors & savings" adds real interest rate

4. Results

While the data about asset, $\mathbf{p}_0$ and $\{\mathbf{x}\}_{t=0}^{t=T}$, suggests the feasible region of volatility of discount factors, the macroeconomic evidence informs their observed volatility. I then could calibrate the asset pricing theory with the real business growth model. I employ a classical representative-agent consumption-based capital asset pricing model (C-CAPM) with *complete* and *frictionless* asset market (Lucas 1978 and Breeden 1979). Investors can freely buy or sell as much of securities as he wishes with zero transaction cost.

The representative investor's problem is to maximize his/her discounted value of expected future utility by deciding how much to consume, how much to save, and what portfolio of assets to hold. At the equilibrium, today marginal utility loss from saving (or invest) is equivalent to discounted marginal utility gain from expected asset's return. It is equated by the stochastic discount factor. With the time-separable CRRA utility, $u(c_t) = c_t^{1-\gamma}/1 - \gamma$, the stochastic discount factor is equal to

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}, \tag{7}$$

where γ is coefficient of relative risk aversion and β is subjective discount factor. When I observe the consumption data, $E(m)$ can be calculated as a function of parameters γ and β.

The volatility of stochastic discount factors are constructed by equation 7, using data of per capita national private consumption expenditure. Figure 3 plots their volatility of as a dashed line. Triangle symbols are plotted at γ increments of 2. The subjective discount factor is assumed to be 0.99 in quarterly basis.

Figure 3. Hansen Jagannathan bound for volatility of stochastic discount factors

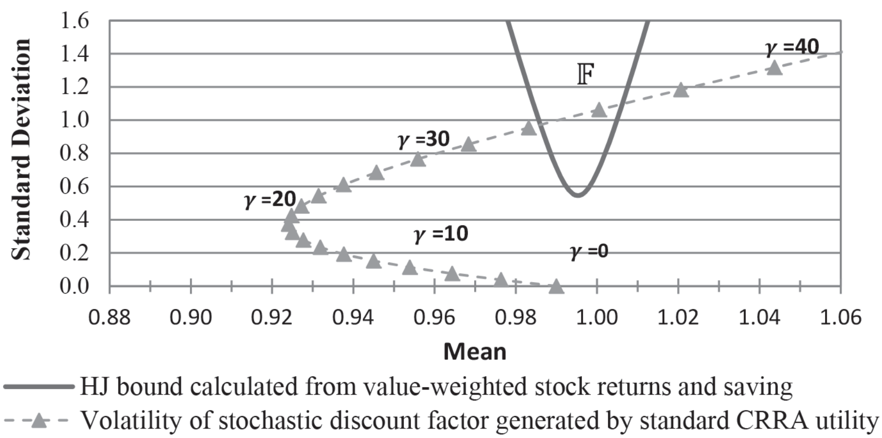


Figure 3 also plots the HJ bound with the quarterly subjective discount factor equal to 0.99. It uses quarterly payoffs of 21 value-weighted portfolios and savings in commercial banks, from 1993-2010. $\mathbb{F}$ denotes the feasible region implied by the bound. The coefficient of risk aversion is required to be around 30-32 to generate the volatility of stochastic discount factors according to the feasible region. The level of risk aversion is seemingly implausibly high, so the situation induces the Asset Pricing Puzzle.

What happens if γ = 30? I employ the basic relation between consumption growth and riskfree rate (if any) as follows:

$$1/R_t^f = E(m_t) = E[\beta(c_{t+1}/c_t)^{-\gamma}]. \quad (8)$$

I assume exclusively here jointly lognormal distribution of riskfree rate and stochastic discount factor (m). It should be noted that this assumption does not dominate the paper, but it is used only to justify the riskfree rate with a high degree of risk aversion. The subjective discount factor and coefficient of risk aversion are assumed to be deterministic process. I take logarithm on both sides of the equation, so the real riskfree rate follows[7]

$$\ln E[R^f] = -E[\ln m] - \frac{1}{2}var[\ln m] - \frac{1}{2}var[\ln R^f] - cov[\ln m, \ln R^f]. \quad (9)$$

While $var[\ln R^f]$ and $cov[\ln m, \ln R^f]$ are both zero, it yields a condition for the riskfree rate as

$$\ln R^f = -E[\ln m] - \frac{1}{2}var[\ln m]. \quad (10)$$

Following the property of lognormal distribution,

$$E[\ln m] = \ln\beta - \gamma E[\ln(c_{t+1}/c_t)], \quad (11)$$

and $$var[\ln m] = var[\ln(c_{t+1}/c_t)]. \quad (12)$$

Then equation (9) becomes

$$\ln R^f = -\ln\beta + \gamma E[\ln(c_{t+1}/c_t)] - \frac{1}{2}\gamma^2 var[\ln(c_{t+1}/c_t)], \quad (13)$$

or $$r^f \approx -\ln\beta + \gamma E[\Delta c_{t+1}] - \frac{1}{2}\gamma^2 var[\Delta c_{t+1}], \quad (14)$$

where Δc_{t+1} is net consumption growth rate.

Quarterly real interest rate is observed as 0.4%, and quarterly net consumption growth rate is 075% with 0.0367% variance. The subjective discount factor is typically close to one. For example, with γ = 30 and β = 0.999 for quarter, I predict the riskfree rate $r^f \approx$ 6.09% per quarter in real term. This is unreasonable to the real world. On the other way, to get the observable interest rate around 0.40% per quarter, the coefficient of risk

[7] Some property about the lognormal distribution follows
$$E(X^aY^b) = exp\left\{aE(lnX) + bE(lnY) + \frac{1}{2}[a^2var(lnX) + b^2var(lnY) + 2abcov(lnX, lnY)]\right\}.$$

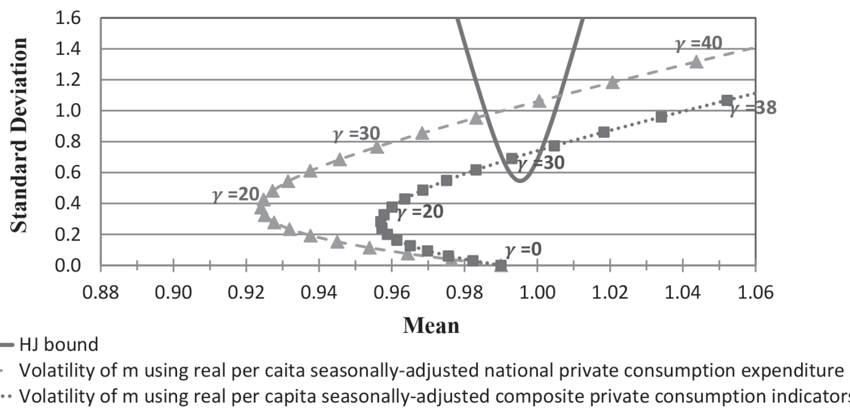
aversion would be only 0.41. Hence the high degree of risk aversion, as $\gamma = 30$, cannot be accepted since it suggests an implausibly high riskfree rate.

5. Robustness Test

I. Other Indicators of Consumption Growth

The result is sensitive to the indicator of a representative agent's consumption. This study hence uses another consumption indicator for robustness test. The Bank of Thailand (BOT) provides monthly data of seasonally-adjusted composite private consumption indicator. The quarterly data is calculated as an average of monthly consumption level during the quarter. It is also available since quarter 1 in 1993. The deflated per capita consumption growth is then computed.

Figure 4. Volatility of **m** using the composite private consumption indicator

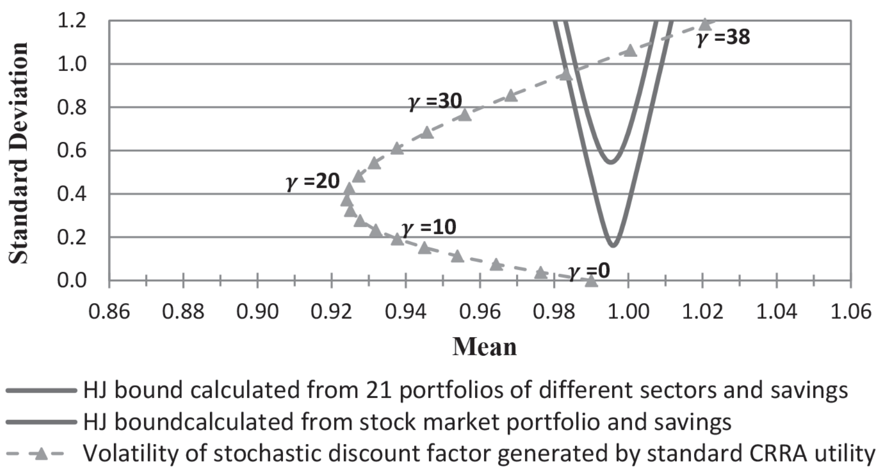


The composite private consumption indicator gives relatively lower average real growth rate of 0.43%. This indicator is also smoother than the national private consumption expenditure. Both real per capita seasonally-adjusted consumption indicators are highly correlated with of 0.964 correlation coefficient level. However, the correlation coefficient of their quarterly growth rates is only 0.428. Figure 4 shows the robustness check of volatility of stochastic discount factors using the BOT composite private consumption indicator. The curves do not alter the result of the asset pricing puzzle. They meet only with implausibly high value of risk aversion.

II. Other Asset Portfolio

Another robustness check is based on number of assets in the market. An investor may consider only two assets including stock market portfolio and saving. It reduces number of assets in constructing the HJ bound to be only two. Mean and variance of the returns on market portfolio are the same as Table 2. The bound is expected to be much lower and wider. The puzzle is then alleviated. Precisely, both curves will become closer. However, Figure 5 shows that the shift of the bound does not help resolving the puzzle.

Figure 5. HJ Bound using two assets – a market portfolio and saving



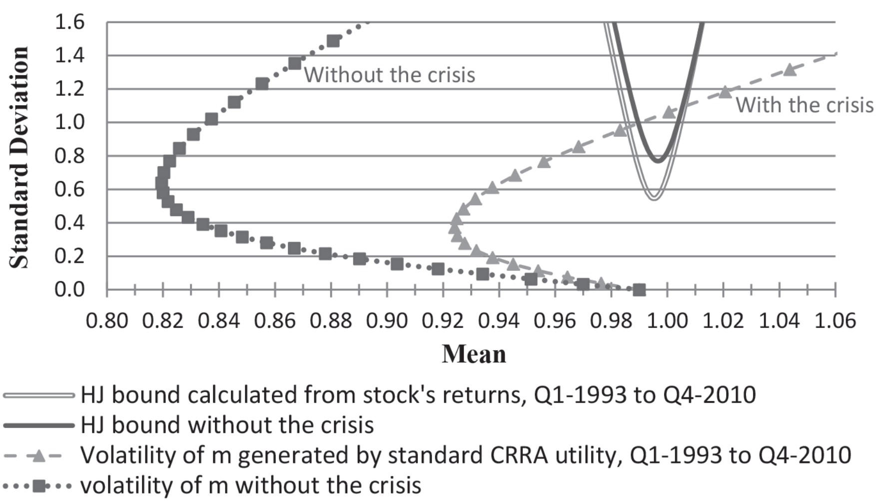
Either alternative indicators of consumption growth or reduction of assets do not end the puzzle. It is still robust.

6. Without the Crisis

One criticism on the calibration of asset pricing puzzle is that most studies use the realization of stock's returns. An investor however considers a chance of a rare economic catastrophe that does not really happened, like Barro (2006). Realized return is thus overestimated, while realized risk is underestimated of the investor's expectation. During the 1997-8 Asian financial crisis, Thailand's real per capita consumption level had dropped 1.8% per quarter on average. The market index had dropped from the highest point at 1,754 in January 1994 to only 207 in September 1998. An investor had lost their money almost 90% in a few years. Especially in 1997, the average real rate of stock's return is -19.2% per quarter. The investors had lost 60% of their wealth in one year. A rare economic crisis is hence realized for Thailand's stock market.

This study considers periods during quarter 1 in 1997 to quarter 4 in 1998 as crisis states, and then analyzes its impact on the asset pricing puzzle. I compare the results by excluding the states of the crisis from Q1-1997 to Q4-1998. Figure 6 shows such comparison. The dots are plotted as an increment of γ by two. When taking out the states of crisis, the bound shifts upwards or it more restricts the volatility of **m**. A higher and smoother consumption growth rates suggest a lower mean of stochastic discount factors, so that the volatility curve turns around very slowly. A coincidence of volatility of **m** with the feasible region is then much more difficult.

Figure 6. Volatility of **m** with and without the 1997 Asian financial crisis



If the investor does not consider the possibility of the crisis, and he/she does not take into account such states in forming the expectation, I then construct the volatility of **m** as a red curves. The asset pricing puzzle is severe. In other words, the inclusion of the 1997 Asian financial crisis much helps explaining the puzzle. It induces a higher expected value of stochastic discount factors. However, this study shows that it is not enough to explain the puzzle. The feasible region is still too high. It requires an implausibly level of risk aversion, which brings about the unrealistic riskfree rate, as discussed in section 4.

7. Alternative Discount Factors

Since alternative preference structures produce different discount factors, they may meet the bound with the appropriate level of risk aversion. This study considers two types of utility functions, GEU and habit persistence utility. The former disentangles parameters about risk aversion and inter-temporal substitution. The latter characterizes that an agent's habit persists. Since he/she is accustomed to consumption level, he/she is more risk averse to consumption volatility.

I. The Generalized Expected Utility (GEU)

According to the constant relative risk aversion (CRRA) utility, the elasticity of inter-temporal substitution is often unable to be disentangled from the coefficient of relative risk aversion. It exhibits constant relative risk aversion with $RRA = \gamma$ and constant elasticity of intertemporal substitution $\varepsilon(c) = 1/\gamma$. A high γ implies that individuals want consumption to be highly smooth over states (they dislike risk); they also desire highly smoothness of consumption over time (they dislike growth).

The "generalized expected utility", proposed by Epstein and Zin (1989) and Weil (1989), is a recursive formulation that is characterized by a constant coefficient of relative risk aversion and a constant but distinct elasticity of intertemporal substitution. The risk aversion parameter influences cross-sectional characteristics of expected returns, while the substitution parameter influences time-series behavior of expected returns. The Epstein-Zin-Weil GEU can be defined recursively by

$$U[c_t, E_t(V_{t+1})] = \frac{\{(1-\beta)c_t^{1-\rho} + \beta[(1-\beta)(1-\gamma)E_t(V_{t+1})]^{1-\rho/1-\gamma}\}^{1-\gamma/1-\rho} - 1}{(1-\beta)(1-\gamma)}. \quad (15)$$

where $\gamma > 0$ is the Arrow-Pratt (constant) coefficient of relative risk aversion for static gambles, and ρ represents an inverse of the (constant) elasticity of intertemporal substitution. The parameter ρ is not necessarily positive since the intertemporal consumption can be both substitution or complementary (an occurrence of habit formation). The standard CRRA utility is a special case as $\gamma = \rho$.

The one-period budget constraint can be rewritten more compactly as

$$w_{t+1} = R_t(w_t - c_t), \tag{16}$$

where w_t is the initial wealth. Let $V(w_t, R_{t-1})$ be the value function for consumer with initial wealth w_t when the last observed rate of return is R_{t-1}. The optimization is set to be a recursive problem as follows

$$V(w_t, R_{t-1}) = \max_{c_t} U[c_t, E_t V(w_{t+1}, \lambda_{t+1})], \text{ subject to equation (16)} \tag{17}$$

Denoting $U_{1t} \equiv \partial U_t(\cdot)/\partial c_t$ and $U_{2t} \equiv \partial U_t(\cdot)/\partial E_t V_{t+1}$ as partial differentiation, the resulting stochastic discount factor is (see Weil 1989)

$$m_{t+1} = \frac{U_{2t} U_{1t+1}}{U_{1t}} = \left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}} [R_{w,t}]^{\frac{1-\gamma}{1-\rho}-1}, \text{ for } \rho \neq 1, \tag{18}$$

where a subscript w denotes a portfolio of aggregate wealth. I use the value-weighted return on stocks in the SET market as a proxy of the portfolio of aggregate wealth.

Figure 7. HJ bound and the volatility of stochastic discount factors generated by the generalized expected utility function

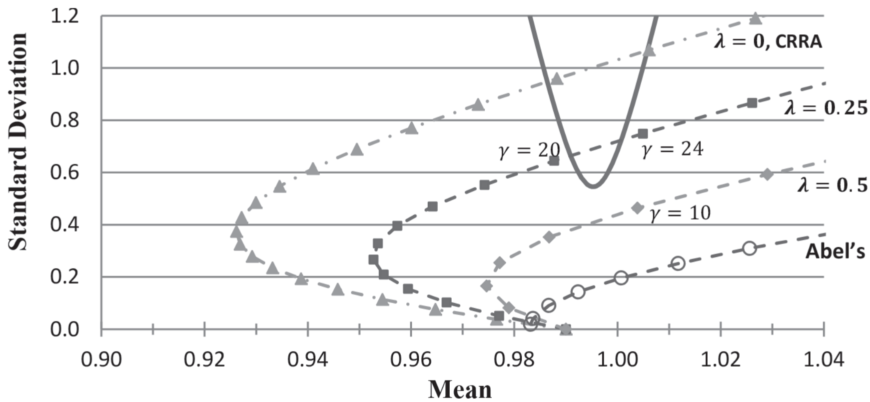


Figure 7 shows the observed volatility of the stochastic discount factors generated by the GEU with different levels of parameter ρ as different dashed lines. All symbols are plotted at γ increments of 2. The volatility of stochastic discount factors to meet the region $\mathbb{F}$ requires the parameter ρ is greater than 5. The high value of ρ implies low elasticity of inter-temporal substitution or low responsiveness of agent to the change in interest rate. For example of $\rho = 10$ case, it requires the coefficient of risk aversion to be around 24, while the $E(\mathbf{m})$ is around 0.995. It still requires an implausibly high degree of risk aversion.

II. The Habit Persistence Utility

Intuitively, an agent is accustomed to his/her own standard of living, and then desires for higher consumption in the future. The habit persistence utility exhibits adjacent *complementarity* in consumption; that is, an increase in consumption increases marginal utility of consumption at adjacent dates relative to the marginal utility of consumption at distant ones. This structure models the agent extremely averse to consumption risk.

Denoting H_t the habit persisting to the period t, the habit persistence utility can be represented as

$$u(c_t, \mathrm{H}_t) = \frac{(c_t - \mathrm{H}_t)^{1-\gamma}}{1-\gamma}, \tag{19}$$

like **Constantinides (1990)** and **Campbell and Cochrane (1995)**. γ is utility curvature, but not the coefficient of relative risk aversion. The coefficient of risk aversion is changing, depending on the levels of individual's consumption and habit. If H_t is held fixed as c_t varies, the *local* coefficient of relative risk aversion is

$$RRA = -\frac{c_t \partial^2 u_t / \partial c_t^2}{\partial u_t / \partial c_t} = \frac{\gamma c_t}{\mathrm{c_t} - \mathrm{H_t}} = \frac{\gamma c_t}{\mathrm{S_t}}, \tag{20}$$

where $S_t \equiv c_t - \mathrm{H}_t$ denotes consumption surplus at time t. The local coefficient of risk aversion is hence possibly high especially at a low level of surplus. The intuition is that an investor may be very risk averse when his/her consumption level is low relative to historical consumption, and when consumption surplus is already high, he/she becomes relatively less risk averse.

This study considers habit as *external*, or "catching up with the Joneses" formulation. An individual's habit level depends on the history of aggregate consumption, i.e. $H_t \equiv \lambda C_{t-1}$, where λ is habit parameter, and C_{t-1} is aggregate per capita consumption level in period $t-1$. An investor observes the peer's consumption level in last period and considers it as his/her benchmark. The habit persistence utility function is then

$$u(c_t, H_t) = \frac{(c_t - \lambda C_{t-1})^{1-\gamma}}{1-\gamma}. \tag{21}$$

The marginal utility of consumption is

$$\frac{\partial u_t}{\partial c_t} = (c_t - \lambda C_{t-1})^{-\gamma}. \tag{22}$$

λ is habit parameter, which is possibly both negative and positive. If the agent is a jealous sort, then λ is positive. The marginal utility of consumption is an increasing function of the aggregate consumption. An increase in aggregate consumption raises the marginal utility of an individual. If individual is patriotic, then λ is negative. The consumption externality is a substitute. The stochastic discount factor is

$$m_{t+1} = \beta\left(\frac{c_{t+1} - \lambda C_t}{c_t - \lambda C_{t-1}}\right)^{-\gamma} = \beta\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}. \tag{23}$$

At the equilibrium of a representative agent model, an individual consumption is equal to the aggregate per capita consumption (since all consumers are identical), or $c_t = C_t$. The equation (21) is rewritten as

$$m_{t+1} = \beta\left(\frac{c_{t+1} - \lambda c_t}{c_t - \lambda c_{t-1}}\right)^{-\gamma}. \tag{24}$$

Another form of habit formation is proposed by **Abel's (1990)**, which is specified as the following form:

$$u(c_t, H_t) = \frac{(c_t / C_{t-1})^{1-\gamma}}{1-\alpha}. \tag{25}$$

An investor compares his/her own current consumption in proportion with peer's consumption in last period. It guarantees non-negativity of the consumption surplus. Since C_{t-1} is fixed at period t, the Arrow-Pratt measure of relative risk aversion is

$$-\frac{c_t \partial^2 u_t / \partial c_t^2}{\partial u_t / \partial c_t} = \gamma, \tag{26}$$

which is constant for all levels of consumption and habit. The marginal utility of consumption holding aggregate consumption unchanged is

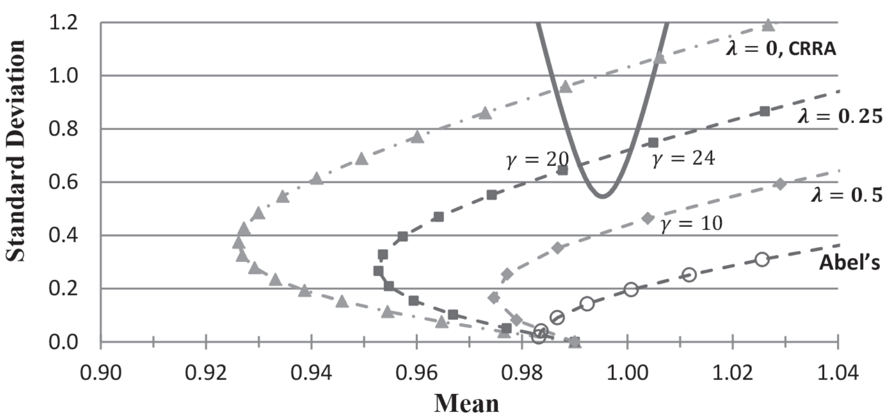
$$\frac{\partial u_t}{\partial c_t} = \mathrm{c}_t^{-\gamma} / \mathrm{C}_{t-1}^{(1-\gamma)}. \tag{27}$$

At the equilibrium, $c_t = C_t$, and the stochastic discount factor is

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{c_t}{c_{t-1}} \right)^{\gamma - 1}. \tag{28}$$

Figure 8 shows the volatility of stochastic discount factors generated by both the Abel's and the Campbell and Cochrane's utility functions with different values of habit parameter λ. The very low value of λ implies similar case as the CRRA utility function. Habit persistence makes stochastic discount factors much more volatile. When I apply the appropriate habit persistence (low value of habit parameter), it helps explaining the puzzle to some extent. Specifically, it requires smaller degree risk aversion in calibration.

Figure 8. HJ Bound and the volatility of stochastic discount factors generated by the habit persistence utility function



Again, an acceptance of $\gamma = 20$ induces a high riskfree interest rate. The relation between consumption growth and riskfree rate is as follows:

$$1/R_t^f = E(m_t) = E\left[\beta\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}\right]. \tag{29}$$

I again assume jointly lognormal distribution of riskfree rate and stochastic discount factor (m). Following the property of lognormal distribution,

$$E[\ln m] = \ln\beta - \gamma E\left[\ln\left(\frac{S_{t+1}}{S_t}\right)\right], \tag{30}$$

and

$$var[\ln m] = \gamma^2 var\left[\ln\left(\frac{S_{t+1}}{S_t}\right)\right]. \tag{31}$$

Then equation (13) becomes

$$\ln R^f = -\ln\beta + \gamma E\left[\ln\left(\frac{S_{t+1}}{S_t}\right)\right] - \frac{1}{2}\gamma^2 var\left[\ln\left(\frac{S_{t+1}}{S_t}\right)\right]. \tag{32}$$

The riskfree rate depends on the surplus growth rate. For $\lambda = 0.25$, the observation suggests logarithmic of surplus growth rate has its mean 0.72% and variance 0.0665%. For example, with $\gamma = 20$ and $\beta = 0.999$ for quarter, I predict the real riskfree rate $r^f \approx -\ln 0.999 + 20 \times 0.72\% - \frac{1}{2}(20)^2 \times 0.0665\% = 0.0120$ or 1.20% per quarter. This is neither reasonable to the real world. However, it notices us that the interest rate is required to be much lower than the CRRA case. Habit persistence model helps much increase the precautionary saving term due to high volatility in consumption surplus, which drives down the interest rate.

8. CONCLUSION

An asset pricing puzzle is detected in emerging stock market as Thailand from 1993-2010. Asset payoffs and consumption growth are observed highly volatile. This study adjusts a calculation of stock payoffs firm by firm including value of offering rights, and finds significant difference to the use of market index. The calibration uses the Hansen Jagannathan bound which is valid with any number of assets and without the riskfree asset. The results show that, with reasonable degree of preference parameters, the asset pricing data is inconsistent with the standard Real Business Cycle model. Precisely, the observed volatility of stochastic discount factors is far outside the feasible region suggested by the Hansen Jagannathan bound.

This study analyzes an impact of the 1997-8 Asian financial crisis on the puzzle. The inclusion of crisis induces a much higher stochastic discount factors. However, it is not enough to resolve the puzzle. It also extends to alternative formula of discount factors generated by different utility functions. The puzzle remains in all cases. Habit persistence alleviates the asset pricing puzzle due to high volatility in consumption surplus. At the same time, it requires lower interest rate through higher precautionary saving.

One shortcoming of this study is a use of aggregate national private consumption indicator. The aggregate growth rate may not be a good representation of the consumption of investors in the SET market. Other indicators or panel survey could help justify the results. The major assumptions include complete and frictionless asset market, as well as homogeneity of agent. A Relaxation of such assumptions could alleviate the puzzle to some extent. For further study, the market frictions; i.e. transaction cost and solvency constraint, and heterogeneity of investors should be applied.

A measurement of expected asset returns, rather than a use of realization, should be tested, see Blanchard (1993) and Fama and French (2001). While the former uses the conditional time-series regression, the latter applies unconditional dividend and earnings growth model. They show that the expected returns are much lower than historical realization. The observable high asset payoffs in the past are unexpected. For further study, this estimation seemingly alleviates both equity premium puzzle and asset pricing puzzle.

References

Abel, Andrew B. (1990). "Asset Prices under Habit Formation and Catching Up with the Joneses", *American Economic Review*, *80*(2), 38–42.

Barro, Robert J. (2006). "Rare Disasters and Asset Markets in the Twentieth Century", *Quarterly Journal of Economics*, August, 823-866.

Bernartzi, Shlomo and Thaler, Richard H. (1995). "Myopic Loss Aversion and the Equity Premium Puzzle", *Quarterly Journal of Economics*, *110*(1), 75-92.

Blanchard, Olivier J. (1993). "Movement in the Equity Premium", *Brooking Papers on Economic Activity*, 2, 75-138.

Breeden, Douglas (1979). "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities", *Journal of Financial Economics*, *7*(3), 265-296.

Campbell, John Y. (1999). "Asset Prices, Consumption, and the Business Cycle", *NBER Working Paper*, 6485.

Campbell, John Y. and Cochrane, John H. (1999). "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior", *Journal of Political Economy*, 107, 205-251.

Cochrane, John H., and Hansen, Lars Peter (1992). "Asset Pricing Explorations for Macroeconomics", *NBER Working Paper*, 4088.

Constantinides, George M. (1990). "Habit Formation: A Resolution of the Equity Premium Puzzle", *Journal of Political Economy*, *98*(3), 519-543.

Epstein, Larry G. and Zin, Stanley E. (1989). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns I: A Theoretical Framework", *Econometrica*, *57*(4), 937-969.

Fama, Eugene F. and French, Kenneth R. (2001). "The Equity Premium", *The Center for Research in Security Prices Working Paper, University of Chicago*, 522.

Gali, Jordi (1994). "Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice and Asset Prices", *Journal of Monetary, Credit, and Banking*, *26*(1), 1-8.

Grossman, Sanford J. and Shiller, Robert J. (1981). "The Determinants of the Variability of Stock Market Prices", *American Economic Review*, 7, 222-227.

Grossman, Sanford J., Melino, Angelo and Shiller, Robert J. (1985). "Estimating the Continuous Time Consumption Based Asset Pricing Model", *NBER Working Paper*, 1643.

Hall, Robert E. (1981). "Intertemporal Substitution in Consumption", *NBER Working Paper*, 720.

Hansen, Lars P. and Jagannathan, Ravi (1991). "Implications of Security Market Data for Models of Dynamic Economies", *Journal of Political Economy*, 99, 225-262.

Hansen, Lars P. and Richard, Scott F. (1987). "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models", *Econometrica*, 55, 587-614.

Hansen, Lars P. and Singleton, Kenneth J. (1982). "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models", *Econometrica*, 50, 1269-1286.

Hansen, Lars P. and Singleton, Kenneth J. (1983). "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns", *Journal of Political Economy*, *91*(2), 249-265.

Kocherlakota, Narayana R. (1995). "The Equity Premium: It's Still a Puzzle", *Federal Reserve Bank of Minneapolis, Discussion Paper*, 102.

Lucas, Robert E. Jr. (1978). "Asset Prices in an Exchange Economy", *Econometrica*, 46, 1429-1445.

Mehra, Rajnish (2003). "The Equity Premium Puzzle", *Journal of Monetary Economics*, 15, 145-161.

Mehra, Rajnish and Prescott, Edward (1985). "The Equity Premium Puzzle", *Journal of Monetary Economics*, 15, 145-161.

Poterba, James A. and Summers, Lawrence H. (1988). "Mean Reversion in Stock Prices: Evidence and Implications", *Journal of Financial Economics*, 22, 27-59.

Reitz, Thomas A. (1988). "The Equity Risk Premium: A Solution", *Journal of Monetary Economics*, 22, 117-131.

Sedthapinun, Thatchai (2000). *C-CAPM and Equity Risk Premium in Thailand*. Thesis, Thammasat University.

Siegel, Jeremy J. and Thaler, Richard H. (1997). "Anomalies - The Equity Premium Puzzle", *Journal of Economic Perspective*, 11, 191-200.

Tesar, Linda L. and Werner, Ingrid M. (1994). "International Equity Transactions and U.S. Portfolio Choice", *NBER Working Paper*, 4611.

Weil, Philippe (1989). "The Equity Premium Puzzle and the Risk-Free Rate Puzzle", *Journal of Monetary Economics*, *24*(3), 401-421.